How Composition Could be Identity

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Abstract

The paper argues that, as far as logic and abstract metaphysics go, the claim that composition is identity could be true, despite well-motivated arguments against the coherence of the claim. The key to answering the arguments is to reject a principle of universal substitutivity of “identical” plural expressions while retaining Leibniz’ Law, that identicals have the same properties, relations, and so forth. We present a metaphysics and semantics that naturally motivates our rejection of universal substitutivity, and show how the resulting conception of collective identity can be used to analyze mereological notions.
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1 Introduction: Ontological Commitment

In *Parts of Classes*, David Lewis praised mereology for its “ontological innocence” as well as for its intuitiveness and ability (with certain assumptions in place) to give a kind of metaphysical foundation for (the less intuitive, in his view) set theory. Ontologically commit yourself to some things one by one, or commit yourself to their mereological fusion; it’s the same commitment either way. Thus, commitment to a fusion of some things is no further commitment than commitment to each of them. This claim about ontological commitment seems to require an amazing metaphysical claim: for the many to compose the one fusion is just for the many to be (in the sense of identity) the one fusion.

There is something attractive and plausible in this idea, but serious problems loom. Lewis himself was quick to back off of a literal reading of the identity claim, and retreated to the position that the special relation between the many and the one, namely composition, is not identity itself, but rather something very much like it.

But if we give up the claim that composition is identity, it is not clear that we can keep the claim that mereology is ontologically innocent. Setting aside Lewis’ other theoretical jobs for mereology, I believe that there is more logical room for the view that composition is identity than Lewis sees. This paper will try to make the case.

The bulk of this paper will answer an argument that may be implicit in Lewis’ remarks, but is given a more focussed statement in other authors. Before proceeding, however, it will be helpful to sketch an application of our general outlook, an application that shows why mereology is indeed ontologically innocent.

Suppose that Alfred ontologically commits himself to Tom (whom Alfred also knows to be a cat), and to Jerry (known to be a mouse). Suppose Alfred also knows that no cat has a mouse as a part, and no mouse has a cat as a part. Then the mereological fusion “Genie”, of Tom and Jerry is neither a cat nor a mouse, since Genie has Tom as a part and has Jerry...

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as a part. As Byeong-Uk Yi argues in [18], it appears that the fusion Genie is a genuinely new thing, and there is no obvious reason why Alfred’s stated commitments should also commit him to it. Why should we think that ontological commitment to Tom and to Jerry already is commitment to something that is neither a cat nor a mouse? We proceed in stages.

First, we claim that Alfred is committed to three different ways that the property of *weighing ten pounds* could be instantiated: it could be had by Tom; it could be had by Jerry; and it could be had by Tom and Jerry collectively. These are three logically distinct possibilities; call them $P_1$, $P_2$, and $P_3$. Note that $P_3$ is not a truth-function of $P_1$ and $P_2$, but a third basic possibility. Similar remarks go for other properties; in principle, there are now three ways they can be had. And something similar goes for two-place relations: there are three ways a relation might be born, on either side, making for (at least) nine ways a relation might be instantiated. (It may be helpful to say something now about how ‘collectively’ in ‘Tom and Jerry collectively are such and such’ is to be parsed. It is not intended to modify the noun phrase ‘Tom and Jerry’ so that we get a larger noun-phrase ‘Tom and Jerry collectively’ with a different semantic value from ‘Tom and Jerry’. Nor is it intended to modify the verb-phrase ‘are such and such’ to form a larger verb-phrase with a different semantic value from that of ‘are such and such’. ‘collectively’ functions semantically to indicate the mode of connection between (the semantic values of) ‘Tom and Jerry’ and ‘are such and such’.

Second, we claim that $P_3$ may be expressed in two different ways: first, by making plural reference to (each of) Tom and Jerry, which Alfred may achieve by ‘Tom and Jerry’, and combining this (act of) reference with collective predication of the property of *weighing ten pounds*; second, by making singular reference to Genie, and combining this reference with (singular) predication of the same property.

Third, we claim that it is sufficient for ‘Genie’ to refer to Genie if we stipulate: Let ‘Genie’ refer to Tom and Jerry (collectively). Just as commitment to Tom and to Jerry yields three ways that *weighing ten pounds* can be instantiated, it also yields three ways that ‘Genie’ refers to can be instantiated.

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2The idea that there are three possibilities here is used, by Allen Hazen, to argue that plural quantification is not innocent of commitment to abstract objects like sets (*pace* Boolos)[2]. Hazen’s point seems to me to be at the heart of the matter, though my use of it is quite different from his.
Our stipulation makes ‘Genie’ bear the reference relation in the third way, to Tom and Jerry collectively. It refers to Tom and Jerry, but not to either of them.

Fourth, we note that whether or not Alfred chooses to make such a stipulation, his commitments make it available to him. Thus, unless his singular quantifiers are for some reason stipulated to be restricted, e.g., so that they range only over cats and mice, he may truly say “There is something that is neither a cat nor a mouse”\(^3\) (Again, the property of being a cat and the property of being a mouse will each be instantiable in three ways, and similarly for the property of being neither a cat nor a mouse.) Speaking with his quantifiers wide open, Alfred may truly utter this sentence.

Finally, we note that if Alfred were to protest that he can assure us that he is not ontologically committed to a single thing that is neither a cat nor a mouse, we would reply that he may be right, but that we have here the makings only for a verbal dispute about how to express his commitments. If by “single thing” he means “either a cat or a mouse”, then yes, he is not committed to such a thing. If he means “thing that falls under a natural kind”, then, probably he is right. But if he means “possible objectual component of a fact”, then he is wrong.

Commitment to Tom and to Jerry turned out to be commitment also to Genie. Genie is, in one sense, another thing: it is not identical with Tom, and it is not identical with Jerry. But in another sense, Genies is not another thing: it is identical with Tom and Jerry. We now note that (for all we have said) Alfred is not committed to a fourth thing, not identical with any of Tom, Jerry, or Genie. Though he is committed to a thing that is identical with Tom and Genie (collectively), that thing just is Genie. It was crucial to our argument that for Tom and Jerry collectively to bear a property is not for either one of them to bear it. This was guaranteed by the joint truth of the facts that Tom is a cat, that Jerry is a mouse, that no cat is part of a mouse, and that no mouse is part of a cat. But for Tom and Genie collectively to bear a property is nothing more or less than for Genie to bear it. Why? (1) Genie is identical with Tom and Jerry (collectively), and hence (since (1a) Tom and Genie are identical with Tom and Genie), (2) Tom and

\(^3\) To connect quantification with the notion of a bound variable, and truth-conditions with variable assignments: in Alfred’s situation, he is committed to the possibility of a singular variable bearing the is assigned to relation to Tom and Jerry collectively (while not bearing it to either of them).
Genie are identical with Tom, Tom and Jerry; but (3) Tom, Tom and Jerry are just Tom and Jerry, and so are identical with Genie. (The inference from (1) and (1a) to (2) is justified by Leibniz’ law; the inference from (2) to (3) is justified by a general “absorption” principle about plurals.)

There are many objections that might be made to our argument about Alfred’s commitment. There may be a sense of “ontological commitment” that is highly deferential to the subject’s belief system, so that one might be committed to Hesperus without being committed to Phosphorus; or committed to water without being committed to hydrogen; or committed to Tom without being committed to Tom’s brain or to any carbon atoms; or committed to the atoms in the universe, including Tom’s atoms, without (despite the fact that Tom’s atoms are arranged, as they are, “cat-wise”, in their current environment with its actual history) being committed to Tom. But surely there is another sense of commitment, (and, if you like, of “what your quantifiers range over”) on which some of these pairs of commitment and non-commitment are not possible, even though the subject may not know it, may not be able to know it \textit{a priori}, indeed may not be able to know it at all. Many of the possible objections can be met simply by making this distinction.

Another kind of objection can be made: Someone we might call a “singularist mereologist” might accept that Alfred is committed to Genie (in the relevant sense of commitment) but reject our account of why this is. He might suggest that formal mereology is a body of necessary truths, and that this is why commitment to Tom and Jerry necessitates commitment to their fusion, Genie. We could agree; the disagreement would be over our suggestions, in particular, that it is sufficient to refer to Genie that one refer to Tom and Jerry collectively, and, in general, (taking for granted a notion of “entering into a fact in a manner”) that for Genie to enter into a fact in some manner is no more or less than for Tom and Jerry to enter into it, in that manner, together. The “singularist” will insist that it is illegitimate to regard Genie as anything but a single object to which Tom and Jerry each bear the primitive \textit{is part of} relation. Our dispute is thus about whether the singularist notions of \textit{part}, \textit{composition}, and \textit{fusion} can be explained in terms of identity and plurality.

\footnote{Ted Sider takes a different approach from ours when he gives a limited defense of the claim that composition is identity in \cite{Sider}. He takes the notion of composition to be defined in terms of the notion of part, and then takes the claim that composition is}
throughout the rest of the paper, but here is a taste of it. What it is for Tom to be part of Genie is for Genie to be identical with Tom and Genie (collectively, not each). The fusion of some things is simply that thing which is identical with them. Formal properties of the relevant part-whole relation, such as transitivity and anti-symmetry will be explained or illuminated by our analysis. Consider anti-symmetry. Suppose $a$ is part of $b$ and $b$ is part of $a$; it will follow that $a$ is $b$. (By def. of part) $b$ is $a$ and $b$, and $a$ is $b$ and $a$; but (by the natural light) $a$ and $b$ are $b$ and $a$; so $a$ is $b$.

The problem: unrestricted substitution unacceptable

Intriguing as the thesis that composition is identity might be, there is a logical difficulty at the very heart of the matter. Here is an argument that shows the kind of problem we face:

(0) Tom and Jerry are animals. Premise
(1) Tom and Jerry compose Genie. Premise
(2) Tom and Genie compose Genie. From (1), logic of composition.
(3) Composition is identity. Suppose for reductio
(4) Tom and Jerry = Genie From (1) and (3)
(5) Tom and Genie = Genie From (2) and (3)
(6) Genie are animals. (4) and (0), “substitution”
(7) Tom and Genie are animals. From (5) and (6), “substitution”
(8) Genie is an animal. From (7)

This argument has an unacceptable conclusion (since Genie is a mere fusion of two animals, not itself an animal), and the only suspicious premise is the thesis that composition is identity. Thus the proponent of composition as identity must explain how the reasoning goes wrong. The mistake, I argue, is the assumption of a substitution principle that would validate the inferences from (4) and (0) to (6) and from (5) and (1) to (7). I accept all the other steps in the argument. One might try quarreling with the inference from (7) to (8), exactly on the grounds that Genie is not an animal! This seems to me to be a mistake. A parallel move would be required to ban the inference from ‘Tom is one of Jerry and Genie’ (which can be ac-
quired from logical truths and the kind of “substitution” used above) to ‘Tom is Jerry or Tom is Genie’[5]. But this inference, like the inference from (7) to (8), are basic to the logic of plural language[6].

The statements on lines (4) and (5) are ambiguous, between distributive and collective readings. (4) could mean that each of Tom and Jerry is identical with Genie, or it could mean that Tom and Jerry collectively are identical with Genie. Only the latter follows from (3), but it does not support the inference from (0) to (4). In general, the mistaken form of substitutivity assumes that for any context \( \phi \)

1. (collectively) are identical with 2. (collectively)

\[ \therefore \phi(1) \text{ if and only if } \phi(2) \]

is valid[7].

Let us call this principle the “Universal Substitutivity of Collectively Identicals” or USCI for short. (For the purposes of this paper, we may ignore counter-examples to USCI that depend on traditional alleged sources of the failure of the substitutivity of identicals, like ‘believes that; and quotation marks. Take the ‘Universal’ in USCI to be already qualified appropriately for whichever of these alleged sources you think are genuine.) If this principle is correct, then composition is not identity, for, whenever some things, referred to by 1, compose some thing (or things) referred to by 2, we should expect to be able to substitute 2 for 1 in “normal” contexts. But this leads to unacceptable results; contradictions even.

Distributive and collective, first pass

We have made use of a distinction between two logical aspects of plural constructions that is commonly made, that between distributive and collective, first pass.

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5 Arguments like the one here are used by Yi (in [18]) against the claim that composition is identity. Similar arguments can be found in Sider [11]. Sider focusses on plural quantification, and plural variables. Lewis himself makes some remarks suggestive of this argument [6].

6 One result of rejecting such inferences (in order to defend composition as identity) is that much of the interesting and intuitive power of plural quantification goes out the window (with the bathwater). Cf. Sider [11].

7 Where the substituends for 1 and 2 are appropriate—roughly, are “referential” plural noun phrases like ‘John and Paul’, ‘they’, and, perhaps, ‘the students’. (Referential, as opposed to quantificational, as ‘all men’ and ‘some men’ are.)
predications involving plural nouns. Both aspects are hard to fit into the straightjacket of first-order logic, and thus many of us will be on relatively unfamiliar ground when we think about them. I think they are presently not fully understood.

A paradigm of distributive predication is ‘Tom and Jerry are animals’. Intuitively, this somehow distributes the property of being an animal (or the predicate ‘is an animal’) to Tom and to Jerry. ‘Each of Tom and Jerry is an animal’ seems to be logically equivalent, and the ‘each of’ somehow indicates the distribution. A paradigm of a collective use is ‘Tom and Jerry (together) weigh ten pounds’. This is not distributive, because there is nothing equivalent with it that predicates ‘weigh ten pounds’ only of Tom and of Jerry. (Of course, there is ‘The sum of Tom’s weight and Jerry’s weight is ten pounds’ which involves only singular terms and predicates, and might be suggested to express the “real fundamental fact” underlying the collective predication, and thus to reveal the true semantic form of the original. This suggestion, taken at face value, is incorrect. Let ‘Head’ name Tom’s head. The suggestion gives the wrong prediction, that Tom and Head collectively weigh seven pounds only if the sum of Tom’s weight and Head’s weight is seven pounds.)

It is not always clear whether to classify a given plural predication as collective or distributive; we will return to the issue of classification in section 3.1. For now, the point is to be clear that identity can be predicated collectively (pace Peter van Inwagen).

We can make sense of the sentence

John and Paul are heavier than George.

The sentence

John and Paul are identical with George.

has exactly the same (surface-)grammatical form. We can easily grasp collective and distributive readings of the first, and we can easily grasp a distributive reading of the second: it simply says what is said by ‘John is identical with George and Paul is identical with George’. (Or, if it doesn’t say

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8 In [16], van Inwagen asks: “what kind of syntactical sense is there in taking either ‘is’ or ‘are’ and putting a singular term or variable on one side of it and a plural term or variable on the other?” (p. 211 of [16]) He then argues that no answer can be given. His arguments are various, but I suggest that they can all be answered in the framework advocated here.
exactly the same thing, it says something logically equivalent with this.) The collective reading of the identity sentence is, perhaps, strange at first, but you get used to it after a while. For example, there are fairly obviously true collective identity sentences, like ‘John and Paul (collectively) are identical with John and Paul (collectively)’ and ‘John and Paul (collectively) are identical with Paul and John (collectively)’. These appear to have the same grammatical form as the readily intelligible ‘John and Paul (collectively) are heavier than George and Ringo (collectively)’. Note that our use of the word ‘collectively’, usually inserted in parentheses, is meant simply to disambiguate, to rule out a distributive reading.

Composition as identity motivates the rejection of USCI; but, of course, USCI seems to be justified by Leibniz’ Law: identicals have all the same properties, enter into all the same relations, etc. I do not reject Leibniz’ Law, of course, so I must explain why this seeming is mere seeming. The next two sections sketch the central metaphysical and semantic ideas that motivate us to reject USCI (while keeping Leibniz’ Law). They also lay foundations for understanding how composition could be identity.

2 Plurals, properties, and relations

We now motivate a picture of properties and relations and the metaphysics of collective property-bearing that will help us to see what the correct plural form of Leibniz’ Law really amounts to.

This will help us to answer the linguistic question: for what kind of context $\phi$ is the form

$$\begin{align*}
\circled{1} \text{ (collectively) are identical with } \circled{2} \text{ (collectively)} \\
\therefore \phi(\circled{1}) \text{ if and only if } \phi(\circled{2})
\end{align*}$$

valid?

Fixed arities for the “multigrade”

It is crucial to our conception that, in short, all properties and relations have fixed “arities”, despite being able to admit more than one thing (simultaneously) in an argument place when predicated collectively of many things. This is the form of collective property-bearing; purely collective
predication occurs when a predicate that expresses a property is predicated of some things in such a way as to “load” those things into the argument-spot of the property.

To flesh out the picture, we begin by taking for granted that there are objects, properties, and relations that somehow come together to form (atomic) facts and propositions, and that complex facts and propositions are somehow formed out of these atomic facts. We take for granted that there is a straightforward, systematic correlation between atomic entities and the “atomic” sentences of a language with names for objects and predicates for properties and relations. We take for granted that, at least up to a certain point, there are further systematic correlations between “complex” entities and more logically complex sentences of a language that includes “atomic” sentences. We assume the correlation is plausible for a classical first-order language.

What happens when we introduce considerations of plurality? In the metaphysics, we greatly increase the number of atomic propositions. For any property, not only is there, for each object, a proposition formed by predicating it of that object; there is also, for any objects, a proposition formed by predicating the property of those objects. Thus, if there are $n$ objects, then, for each property, there are $2^n - 1$ atomic propositions predicating that property.

Please do not replace the property with a simulation of it: it is not a set of objects, nor a set of sets of objects, nor a function from possible worlds to sets of objects or sets of sets of objects. It is a metaphysical primitive. Perhaps it could be reduced, in the end, to some such thing; but not yet. In particular, do not take us to be suggesting that a property is a set of sets of objects.

Similar remarks go for relations. For any two-place relation, either place can take any objects; hence, if there are $n$ objects, then for any re-

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9We thus agree with Byeong-Uk Yi that some properties can accept many things “as such” as arguments, and generally that some “blank spots” in relations can accept many things (collectively) as arguments. See section 5 of [17]. Unlike Yi, we think that this is so for all properties and “blank spots” of relations.

10This count assumes that the identities of these objects are “independent” in a certain sense. None of them is part of another in the sense of part definable in terms of identity. Using the technical vocabulary given in later sections: we assume that none of them is an i-part of another.
How Composition Could be Identity

Let us turn to language. Consider expanding first-order grammar to include plural complex terms (corresponding, e.g., to ‘John and Paul’) and an apparatus of plural quantification (corresponding to the apparatus in ‘There are some things such that . . . they . . . ’). Suppose that outweigh expresses a relation in a language with the expanded grammar. Consider the sentence

John and Paul outweigh Mary.

(Imagine that Mary is a large guitar amplifier). If we avail ourselves only of metaphysical ideas appropriate for a classical first-order framework, this sentence presents a logico-semantic problem. Whereas the distributive reading of the plural predication reasonably can be thought of as expressing (when true) a conjunction of two atomic facts, the collective reading must be approached differently.

I suggest that we think of the collective reading as expressing (if true) the fact that the outweighing relation holds between John and Paul (collectively, not each), on the one hand, and Mary, on the other. The outweighing relation involved is the very same two-place relation that is involved in the fact that John outweighs Ringo. It is a “two-place” relation, but more than one thing can (simultaneously, so to speak) fill one of its places. The collective mode of predication is what connects (one of the blank spots of) the relation with John and Paul.

This conception has a great advantage over conceptions on which what is really going on in this case involves a three-place relation (or three-place instance or determinate of a multigrade relation).

Suppose we think of ‘outweigh’ as here expressing a three-place relation, that holds among John, Paul, and Mary. Then

John, Paul, and George (collectively) outweigh Mary.

would involve a different, four-place, relation, and

Some men (collectively) outweigh Mary.

\[1^{1} \] The term ‘multigrade relation’ seems to come from Leonard and Goodman. A multigrade relation can apply as if it had any number of blank spots. But what exactly does this mean? Most attempts to make this out have focussed on formal logic and semantics, rather than metaphysics, as in and.
would involve covert restricted quantification over relations; it would say something like

There are some men, and there are some relations, and each of those relations is an “outweighing relation”, and those men bear one of those relations to Mary.

Besides the implausibility of the suggestion of the covert quantification over relations, this approach faces the problem of clarifying the notion of a “outweighing relation,” which would appear to be a new category of property of relations.

Further, consider the difference between (the collective readings of)

John and Paul outweigh George and Ringo.

and

John and Paul and George outweigh Ringo.

If ‘outweigh’ in both examples acts as a four-place relation, it would seem that the very same proposition, one we might represent as

\[ \text{Outweigh}(\text{john}, \text{paul}, \text{george}, \text{ringo}) \]

is being expressed by both sentences. Clearly, the two English sentences are not logically equivalent. Thus it is much better to think of ‘outweigh’ as, once and for all, expressing a two-place relation. We might represent the propositions expressed by the two sentences like this:

\[ \text{Outweigh}(\text{john}, \text{paul}, \\
\phantom{\text{paul}}\text{george}, \text{ringo}) \]

and

\[ \text{Outweigh}(\text{john}, \text{paul}, \text{george}, \text{ringo}) \]

respectively.

On our conception, there is no fundamental distinction between “plural” properties and “singular” ones. There may be, of course, properties that are actually possessed only by metaphysically singular things. But

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12 A metaphysically singular thing would be something that is not identical with some (more than one) things (collectively). We discuss these objects in more detail below.
the propositions formed by predicating such a property exist nonetheless—the point is that they are false. Similarly, there may be properties that are actually possessed, but never by metaphysically single things. Again, the propositions formed by predicating such a property of one thing will simply be false, maybe necessarily false or even logically false, not non-existent.

It is important to note that on our conception, the expansion of the traditional logical-atomist metaphysics to include collective property-bearing is not the first step up a hierarchy. Call the traditional atomic propositions singular, and call the ones we have introduced plural. You can form a plural atomic proposition by predicating a property of some things; but you cannot form a non-singular, non-plural, “plurally plural” proposition by predicating a property of some things$_{xx}$ and some things$_{yy}$. At best, you can form a conjunction of two plural propositions, one about them$_{xx}$ and one about them$_{yy}$, or you can form one plural proposition, the one formed by predicating the property of those things$_{zz}$ (not things$_{es}$!) such that they are all and only the things that are either one of them$_{xx}$ or one of them$_{yy}$. “Plurally plural” talk is metaphysically insignificant. Plurally plural talk is at best a mere verbal code for plural talk, exactly because what it is for two “thingses” to have a property is either (1) for each of them to have it (in which case we have nothing new) or (2) for the things you get when you “put the two thingses together” to have it.

**Leibniz’ Law and substitution**

Identity is a relation, so it, too, can accept any number of things simultaneously in either blank spot. But it is identity. So:

For any things $xx$ and any things $yy$,

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13 This is not to deny that there could be a term $t$, an allegedly plurally plural term, that bears a relation (call it ‘reference’) to some things (collectively) and some other things (collectively) and maybe still other things. In fact, that is exactly how we think of grammatically plural terms! The point is that any “property” that looks like a property of thingses is just a property of things (a property that can be had by a thing or some things collectively) or definable in terms of plural quantification and properties of things.

14 This is not to deny that there could be a hierarchy of properties. Perhaps there are properties of properties, properties of properties of properties, and so forth. If so, one could simulate a hierarchy of orders of plurality with these properties (given certain assumptions about their plenitude). Cf. [9].
if the result of putting \( xx \) (collectively) in one blank spot of the identity relation and \( yy \) (collectively) in the other is true then the result of putting \( xx \) (collectively) into the blank spot of any property, and the result of putting \( yy \) (collectively) into it will have the same truth value.

(Notation: ‘\( xx \)’ is a plural variable.) (Caveat: we used the notion of \textit{truth} and \textit{truth value} here, but since the bearers of these notions are meant to be atomic and other objectual propositions, the notions of \textit{holding} or \textit{obtaining} might be more appropriate. But, since we don’t have a notion of a \textit{holding value}, the formulation of the principle would then be even more labored.)

A similar, more generalized form should hold for relations.\(^{15}\)

We can now say that

\( \{1\} \) (collectively) are identical with \( \{2\} \) (collectively)
\[ \therefore \quad \phi(\{1\}) \text{ if and only if } \phi(\{2\}) \]

is valid when \( \phi \) expresses collective predication of a property.

3 Distribution and failures of substitutivity

Which kinds of plural contexts do \textit{not} express collective predication? The topic here is the relation of (natural) plural language to our metaphysics.

\(^{15}\)A tricky question arises over the identities of objective propositions involving collective plural predication. Suppose that \( xx \) are collectively identical with \( yy \), but not distributively identical (i.e., one of \( xx \) is not one of \( yy \) or vice-versa). Then is the objective proposition expressed by “\( xx \) are collectively \( F \)” the same as the one expressed by “\( yy \) are collectively \( F \)”?

It is tempting to distinguish two levels of objective propositions: on the less metaphysically fundamental level, before the property is actually predicated, so to speak, we have something like \( \langle xx, \text{coll}, F \rangle \), where \textit{coll} indicates the mode of predication to be used, and \textit{each of} \( xx \) is in the proposition; on the more fundamental level, post-predication, we have something like \( \langle F, xx \rangle \) where \( F \) has been connected with \( xx \) collectively. If this distinction is acceptable, then we may say that the pre-predication propositions involving \( xx \) and \( yy \) are distinct, but the post-predication propositions are the same.
Pure distribution

Some plural predications simply distribute a property. On our view of the semantics of plural subjects (on which more shortly), this is to be filled out as follows: such predications distribute a property to each of the things that the (plural) subject refers to. One kind of case is the predication of a grammatically pluralized noun phrase that predicates a property in its singular form. E.g., ‘___ are animals’ is a pluralized form of ‘___ is an animal’, and this singular form seems to predicate a property being an animal. It is highly plausible that ‘Tom and Jerry are animals’ distributes predication of being an animal to Tom and to Jerry. Thus we may plausibly deny that it predicates any property of Tom and Jerry collectively. The objective proposition corresponding to it includes only the one property being an animal (though it involves that property twice). This is not (yet) to assert that there is no property being animals such that some things have it collectively if and only if each of them is an animal. The relevant current points are (1) that there is a plausible account of ‘___ are animals’ that does not appeal to such a property; (2) this account has intuitive appeal independent of the thesis that identity is a kind of composition.

But given that composition is identity, there can be no property such as being animals. This is, in our view, the real upshot of the argument considered at the beginning of the paper. Suppose there were such a property. Tom and Jerry collectively have it. Genie is identical with Tom and Jerry collectively. So Genie has it. So we should be able to say ‘Genie are animals’. This is weird. But, worse still: Tom and Genie collectively are identical with Genie, so we should be able to say ‘Tom and Genie are animals’. This isn’t weird: it is simply false, because we expect it to follow, from ‘t and s are animals’, that ‘s is an animal’. But Genie is not an animal; Genie is a mere fusion of two animals.

We pause to note an independent disadvantage of the account on which ‘are animals’ expresses a property of being animals. On that view, the logical connection between ‘t and s are animals’ and ‘s is an animal’ is accounted for by appeal to a logical-metaphysical connection between properties. Similar connections will be posited for many other property pairs.

16 Here and forward we have a fundamental disagreement with the work of Byeong-Uk Yi. Yi holds that a typical pluralized noun phrase like ‘are animals’ expresses a plural property, and analyzes ‘Tom and Jerry are animals’ exactly as on our model of collective property bearing: a property is predicated of some (thing or things).
being a donkey and being donkeys, for example. On our account, the logical connection between ‘t and s are animals’ and ‘s is an animal’ is analyzed in terms of a single notion of property-distribution: the pluralized form semantically acts to distribute the property expressed by its singular root. This single semantic mechanism explains all connections of the form ‘t and s are Fs’ and ‘s is an F’. Of course, the opposing account can also say that there is a single mechanism at work: ‘are Fs’ semantically involves an operation on properties that takes being F to being Fs. Yet the original logical connections are still being explained by the connections among these properties. I object that those connections are, at best, being explained twice-over, once by a semantic mechanism, and then by a metaphysical one (or, more accurately, by a multiplicity of metaphysical connections). A single mechanism should be enough.

On our account, the property being animals is not involved in the plural sentence or in the logical connection between the two sentences. And, as we saw, given our view of collective identity and predication, there is no property being animals had by some things collectively just in case each of them is an animal. Note that we tried to define being animals using an expression for the property being an animal and, crucially, ‘each of’. Generalizations of the arguments above yield the conclusion that ‘each of ___ is F’ (where ‘is F’ does express a property) does not express a property. We maintain that they express distribution of the one property F.

Let F be any property. Suppose there is a property F∗ such that some things collectively have F∗ just in case each of them has F. Suppose something x has F, and some things yy each have F. Then if (x and yy) are collectively identical with something z, then x and z are collectively identical with (x and yy), hence with some things each of which has F; hence x and z collectively have F∗, and so z has F. Translating collective identity as composition, what we have shown is: the fusion of any Fs has F. We can also show that if F∗ exists, then any part of an F has F.

Putting these two facts together, we see that F∗ can only exist, as defined, if F is a special sort of property: one had by any fusion of any parts of things that have F. Such properties are rare. (Though they may ex-

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17 This is basically Yi’s property-operation of “neutral expansion.”

18 Suppose x is F, and y is part of x. Then x and y collectively are identical with x, and, hence, collectively identical with x and x. Thus, x and y are identical with some things, each of which has F. So x and y collectively have F∗; y is one of them, so y has F.
ist; some candidates are *being a thing*, *being (completely) red*, and *being some water*.)

The generalization points to the root of the problem for the thought that there are properties like *being animals* was supposed to be; ‘each of’ does not express a relation, and hence cannot generally be combined with a property to yield a property. Neither does its dual ‘one of’. Suppose $F$ is a property and suppose there is a property $F^*$ had by some things collectively just in case one of them has $F$. We can immediately deduce that if some things compose an $F$, then one of them is an $F$. We can deduce that everything that has an $F$ as a part is also an $F$. $F$ would have to be a special sort of property; *being a thing* is about the only natural candidate. The definition of $F^*$ in terms of $F$ cannot generally work; the reason is that ‘is one of’ does not express a relation.

**‘is one of’ does not express a relation**

The expression ‘is one of’ is central to the discussions of plural quantification and logic in the philosophical literature. We have seen that, given our perspective on identity, it does not express a relation. This is a natural idea.

It is natural to suppose that ‘Each of John and Paul is a musician’ does not predicate a property, *being each a musician*, of John and Paul, but instead predicates a property, the property of *being a musician*, in such a way as to yield a proposition that is true just in case *each of* John and Paul has the property.

The parallel account of ‘One of John and Paul is a musician’ is plausible: The effect of this is not to predicate a property of *one being a musician* but, instead, to distribute the property of being a musician. The mode of distribution is different than in the example with ‘each of’; it is “disjunctive” rather than “conjunctive”. So ‘One of __ is a musician.’ does not express a property.

Now consider

John is one of __.

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19Suppose $x$ has $F$. Let $y$ be something such that $x$ and $y$ collectively are identical with $y$. (I.e., $x$ is part of $y$.) Then $y$ has $F^*$. $y$ and $y$ collectively are identical with $y$, hence one of $y$ and $y$ has $F$; hence $y$ has $F$. 

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This is equivalent with

One of __ is identical with John.

And we should say the same thing about it: it does not express a property; rather, it distributes the property of being identical with John, and it distributes this property disjunctively. The ‘is’ of ‘is one of’ is the ‘is’ of identity, which expresses a relation; the ‘one of’ somehow “expresses” a logical operation of disjunctive distribution.

So what does ‘one of’ do?

We will give a more detailed discussion of a truth-conditional semantics of plural language in section 5 but it will help to look at one element of our theory now. (In many respects, our account is just like that of Boolos[1].)

We take a grammatically plural term to refer to (each of) many things (except in the limit case in which it is semantically like a singular term, and refers to exactly one thing, e.g. ‘Hesperus and Phosphorus’). This deserves emphasis; it is absolutely central to our view that plural terms are referentially quite different from singular terms. A singular term refers only once—there is exactly one thing to which it bears the refers to relation. We suggest that the truth-conditions for something of the form

s is one of t

(with singular term s and plural term t) are:

One of the things that t refers to is identical with the thing that s refers to; that is (since s is singular):
there is something t refers to and which is identical with something that s refers to.

The truth-conditions need not be, and should not be, given like this:

What s refers to bears the one of relation to what t refers to.

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The truth-conditional description of the role of ‘is one of’ is not meant to compete with what we have said about objective propositions, but to provide a rather different angle on what we take to be, ultimately, the same basic idea about ‘is one of’.
How Composition Could be Identity

Our preferred truth-conditions ascribe a meaning (a semantic function) to ‘one of’ without appealing to a relation at all. And it is worth noting that on our view, “what t refers to” is a potentially misleading locution. A plural term refers to (each of) many things. “what t refers to” should be compared with “what you own”.

Given this understanding of plural reference, it is easy to see how to give the truth-conditional semantics for simple distributive predicates like ‘are animals’. In general ‘t are Fs’ will be true just in case every thing that t refers to has the property expressed by F. Again, no property of being Fs is needed or wanted.

Distributive-collective ambiguity

We need an example of a predicate that simply expresses a property. Let us suppose that ‘weigh 300 pounds’ is such a predicate. (Probably it does not simply express a property: it is composed of three independently meaningful parts. But it is also plausible that it determines a property that could, in principle, be expressed by a grammatically simple predicate. Pretend it simply expresses this property; call the property F.) Consider the English sentence

John and Paul weigh 300 pounds.

This sentence is ambiguous between a distributive and a collective reading. These are given, apparently unambiguously, by

John and Paul each weigh 300 pounds.

and

John and Paul collectively weigh 300 pounds.

We have suggested that the first should be understood to predicate F of each of John and Paul. It does not express an atomic objectual proposition, since F must be predicated twice. Its truth-condition is: Every thing that ‘John and Paul’ refers to is F. The second should be understood to predicate F collectively. Hence, it expresses an “atomic” objectual proposition, in which F is predicated once, of John and Paul collectively. Its truth-condition is: The things, that ‘John and Paul’ refers to, those things collectively bear F.
‘each of’ and ‘one of’ are duals

Here is an observation that may help one to see through the appearance that ‘is one of’ expresses a relation (if it so appears to one). There is a systematic equivalence between sentences involving ‘each of’ and sentences involving ‘one of’, very much like that between ∨ and ∃. For example

John is one of John and Paul.

which might be alleged to express the holding of a supposed is one of relation between John, on the one hand, and John and Paul on the other, is equivalent with

It is not the case that John is non-identical with each of John and Paul.

But surely neither ‘each of’ as used in the latter sentence, does not function by expressing a relation; this is further evidence, we suggest, that ‘is one of’, in the original, does not either.

3.1 Implicit distribution

Sometimes

① (collectively) are identical with ② (collectively)
∴ φ(①) if and only if φ(②)

is invalid—but only when φ does not simply express collective predication of a property. Given that ‘is one of’ distributes the identity relation, and does not collectively predicate some relation, we should not expect substitutivity in contexts that essentially involve ‘is one of’. Similarly for ‘each of’.

Sometimes contexts seem to be logically equivalent with contexts in which the blank spot of the context is the operand for something including ‘one of’ and ‘each of’. So it is with all the straightforwardly distributive predicates like ‘are animals’. If we are right about ‘one of’ and ‘each of’, then these other contexts also should not be regarded as expressing collective predication of a property.

There is not room to go into great detail. Some suggestive examples will have to do:

① are bandmates
seems to be equivalent with

there is a band that Ⓚ each are in

Ⓜ each admire one another

is equivalent with

each of Ⓚ admires a thing only if it is an (other) one of Ⓚ

Sometimes verbs seem to involve implicit quantification over events. In these cases, we may have non-collective plural contexts. E.g.,

Ⓜ conversed

may be analyzable as

there was a conversation, and each of Ⓚ partook in it

Generally, such predicates may not be simply distributive, as ‘are animals’ is. Yet, they are not collective, on our analysis, since they do not express collective predication of a property. Thus we hold that they semantically involve operations on a par with ‘one of’ and ‘each’, along with some properties. Many apparent failures of USCI (Universal Substitutivity of Collectively Identicals) arise from this fact. For example, if we thought that John and Paul are each a fusion of molecules, we would not want to be able to infer, from ‘John and Paul conversed’, that ‘Some molecules conversed’. The analysis above is the skeleton of an explanation of why this inference fails even when composition is identity and, hence, John and Paul (collectively) are identical with some molecules (collectively).

These predicates are neither straightforwardly distributive (like ‘are animals’) nor purely collective. But they seem to be implicitly distributive: in particular, they distribute a relation to an implicit object: e.g., ‘They converse’ says that each of them (and no one else?) bears the is a party to relation to some one event. We seem to require a three-way distinction

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21I believe that this observation will greatly help the “nihilist” to respond to Gabriel Uzquiano’s arguments in [15]. Uzquiano argues that the nihilist—one who believes that all things are (mereological) atoms—cannot give plausible paraphrases of certain English sentences involving plural predicates and quantifiers. If the nihilist gives paraphrases that bring the ‘one of’ constructions to the fore, and then applies our semantics (below), he can meet Uzquiano’s challenge.
between plural predications: those that are straightforwardly distributive (as in ‘each of _ is an animal’, ‘are each an animal’ or ‘are animals’), those that are relationally distributive (like ‘are class-mates’ and ‘converse’), and those that are purely collective (like ‘weigh 100 pounds’ when it is predicated collectively). (Plausibly, the two kinds of distributive predication are species of a genus, while collective predication is another genus.)

Sometimes it is not clear in which category to put a given predicate: e.g., ‘__ surround the building’. Our view predicts that the question is settled by whether the inference from

they surround the building

to

their fusion surrounds the building

is valid. If so, it is a purely collective predicate; if not, then it is relationally distributive. I tend to think it is ambiguous, and can be used to express either: (1) a collective, stative surrounding: a state they enter into “as one”, as a moat might surround a building; or (2) a distributed and active surrounding: an event each of them enters into as an independent (partial) agent; as a team of soldiers might, following an elaborate plan, surround a building. The fusion of the soldiers does not (thereby) surround a building, in this sense. There may also be a collective and active sense, in which an anaconda might surround its prey. Some humans—a mob—might surround a building in this way (by accident); in this case, their fusion does surround the building.

Fred Landman, in the context of a neo-Davidsonian theory of events as implicit arguments of verbs, makes a distinction among plural predications that appears to me to be similar to my distinction between purely collective and implicitly distributive predications. Roughly, purely collective predications propose that the plural subject fills a single thematic role of the predicate, while our implicitly distributive predications propose that the plural subject fills a non-thematic “plural” role. See especially section 5.4 of [4]. Much of his semantics for plurals, I suspect, can be seen as a special case of ours, without the metaphysical underpinnings of composition as identity and the special attention to the identity predicate.
4 Collective reference, collective identity, and composition

We have now laid the metaphysical cornerstone for our defense of composition as identity. We have also given a significant part of our semantic picture, enough to justify our rejecting the Universal Substitutivity of Collective Identity, so that we may escape the difficulties that flow from it: distributive predications do not simply predicate properties, so contexts that semantically involve distribution need not—as far as metaphysics goes—support substitution. We now fill in another piece of the semantic picture: collectively multiple reference. Questions about the nature of collective identity and its relation to constitution naturally arise along the way. As we discuss them, we will introduce, as “axioms”, our most contentful assumptions. We also will make heavy use of Leibniz’ Law, which we may now make explicit as:

Substitution  For any terms t and s:
\[
\text{from } \left[\tau \text{ collectively are identical with } s \text{ collectively}\right] \\
\text{and } \left[\phi(t)\right] \\
\text{one may infer } \left[\phi(s)\right] \\
\text{where } \left[\phi(s)\right] \text{ arises from } \left[\phi(t)\right] \text{ by replacing one or more occurrences of } t \text{ that are arguments for collective predication in } \left[\phi(t)\right].
\]

Assume ‘Tom’ refers to the cat Tom, ‘Jerry’ to the mouse Jerry. What does it mean for a term t to refer to Tom and Jerry? Answer: it could mean either of two very different things. (1) It could mean that t refers to each of them: t refers to Tom and t refers to Jerry. Call this distributively multiple reference. (2) It could mean that t refers to Tom and Jerry collectively, without implying that it refers to either one of them. Call this collectively multiple reference. Distributively multiple reference makes a term semantically plural. For example: ‘Tom and Jerry’ refers distributively multiply; it refers to each of Tom and Jerry. Collectively multiple reference does not make a term semantically plural. Let ‘Genie’ refer to Tom and Jerry collectively and to nothing else. Then ‘Genie’ does not refer to Tom, nor to Jerry. ‘Genie’ is a singular term, both grammatically and semantically. (It is possible for a term to be grammatically plural, but semantically singular: ‘Hesperus and Phosphorus’ is like this.)

There is another way to look at the term ‘Genie’. It refers to Genie, and to nothing else. This is not to deny that it refers to Tom and Jerry (col-
lectively). The italicized clause means: every thing referred to by ‘Genie’ is identical with Genie, and (therefore) any things referred to by ‘Genie’ collectively are identical (collectively) with Genie. Thus, it requires that Genie is identical with Tom and Jerry. Not identical with each of them, of course; identical with them collectively. Put Genie in one of the slots of the identity relation, and put Tom and Jerry in the other: the result holds.

Observe that we may say that there are some things xx such that ‘Genie’ refers to xx collectively. Since ‘Genie’ refers to Genie, it appears that we may also say that there is some thing x such that ‘Genie’ refers to x. This suggests a principle about collective identity: we call it the axiom of Comprehensive Collective Identity (CCI). It is:

\[
\text{CCI} \quad \text{For any things, they are collectively identical with some thing.}
\]

Any things have a singular collective identity. This seems a fairly natural assumption to make, if it is admitted that sometimes, some two or more things are collectively identical with some one thing.

One might resist, and suggest that it is never the case that some things (more than one) are identical with some thing. We have three arguments to make in reply: the argument from objects as proposition-components, the argument from bookkeeping and the argument from infinite descent. None of these arguments has the character of a proof; we don’t call CCI an axiom for nothing. But the arguments may dissolve some resistance.

The argument from proposition-components simply says that, an object is nothing more or less than an objectual component of a proposition. Given some things, given our metaphysics of collective property and relation bearing, those things figure in basic objectual propositions in exactly the way that a single thing does.

The argument from bookkeeping recapitulates and generalizes what we have just said, above, about ‘Genie’. For any things, it is logically possible to refer to them collectively (while not referring to anything else). If a term ‘a’ refers to them collectively (and to nothing else) it behaves as a singular term (classically conceived): e.g., for any property term F, the sentence \( \forall a F(a) \) is associated with a unique basic objectual proposition, one that arises from predicating the property expressed by F of the objects a refers to collectively. Similarly, for any variable, it is logically possible for it to be assigned to them collectively, and it will then behave exactly like a classically conceived singular variable under an assignment. Hence, there
is a logically possible variable assignment that will satisfy “x is identical with a”; hence, “There is some thing x identical with a” is true. Why not descend and disquote? To say that there is such a thing as a (and it is such-and-so) simply indicates that one has chosen a certain notation for talking about what properties and relations they collectively bear. In so doing, one has sealed them off from distributive predications.

One might resist the argument from bookkeeping with the thought that such a notational choice is, even if convenient for some purposes, metaphysically perverse: singular terms and variables should stand for singular things, never for many singular things collectively! The argument from infinite descent has two dialectical steps of reply. Step one: it may be that (some of) the proper names and singular variable (-analogues) of ordinary speech already do collectively refer in the manner envisioned. It is plausible that “There is a stack of books on the table” involves satisfaction of ‘is a stack of books on the table’ by some books collectively. Now we imagine a reply: “So ordinary language is (or may be) perverse! In a metaphysically rigorous language, we would not tolerate this.” Step two: it is not certain that such a language is available. For it may be that every (physical) thing is like the stack of books: to speak of it just is to speak of many things (collectively). It is identical with some things (more than one) (collectively).

So much for the arguments meant to loosen resistance to Comprehensive Collective Identity. Given CCI (and Leibniz’ Law) we have the result that: for any things xx, there is a thing x such that for xx to collectively bear a property or relation is for x to bear that property or relation. Incidentally, this allows us to “reduce” all collective predications to collective predications of identity; instead of predicating F of xx collectively, simply predicate F of the unique x such that xx collectively are x. (Since it appears that all non-collective plural predications can be reduced to predication of “is one of” and singular predications, these two reductions could be combined so that all plural predication whatever is reduced to singular predication plus plural predication of “is one of” and “are” (interpreted collectively at one argument).)

Now note that some terms feature both distributively multiple and collectively multiple reference. ‘Tom and Genie’ refers distributively multiply, since it refers to Tom and refers to Genie, and Tom is not identical with Genie. It refers collectively multiply, also, since it refers to Genie, and to refer to Genie is to refer to Tom and Jerry collectively. Note that ‘Tom’
refers to something that ‘Tom and Jerry’ refers to, but that ‘Genie’ does not refer to, namely Tom. This is how ‘Tom is one of Tom and Jerry’ can be true, while ‘Tom is one of Genie and Jerry’ is false. Yet ‘Tom and Jerry (collectively) are identical with Genie’ is true. ‘Tom and Genie (collectively) are identical with Genie’ is, therefore, also true.

This last inference employs an instance of a principle

For any thing $x$, and any thing $y$, if there is some thing $z$ such that $x$ is identical with $y$ and $z$ collectively, then $x$ is identical with $x$ and $y$ collectively.

This principle may be derived from our next axioms:

**Absorption** For any thing $x$, $x$ is identical with $x$ and $x$ collectively.

and

**Symmetry** For any thing $x$ and any thing $y$, $x$ and $y$ collectively are identical with $y$ and $x$ collectively.

The derivation is as follows. (All plural predications are to be read collectively.) Suppose $x$ is identical with $y$ and $z$. Then (substituting) $x$ and $y$ are identical with $y$ and $z$ and $y$. By Symmetry and substitution, $y$ and $z$ and $y$ are identical with $y$ and $y$ and $z$. By Absorption, $y$ and $y$ and $z$ are identical with $y$ and $z$. So we have a chain of identities starting with $x$ and $y$ and ending with $y$ and $z$; by the transitivity of identity we have that $x$ and $y$ are $y$ and $z$. Using our supposition and substitution, we get that $x$ and $y$ are $x$.

Questions: How many things does ‘Genie’ refer to? How many things does ‘Tom and Jerry’ refer to? Answers: depends what you mean. If you mean to ask how many “times” these terms refer, your questions are well-defined, and the answers are: definitely and uniquely one for ‘Genie’; two for ‘Tom and Jerry’. If you mean to ask how many things there are such that ‘Genie’ (or ‘Tom and Jerry’) refers to them collectively, your questions are (somewhat) ill-defined, and the answers depend on some things. On one reasonable way of refining your questions, it is guaranteed that the answers will be the same. We will explain our answers more precisely in a moment; first we must get a little clearer about the scope and nature of the composition aspect of collective identity.
4.1 Scope of the thesis that composition is identity

There are two ways one could take the thesis that composition is identity: it could be the all-encompassing thesis that whenever composition occurs, it is identity; or it could be the more modest claim that identity is one kind of composition. I find the latter view much more plausible. As I will suggest later, it is plausible, though not certain, that Classical Mereology is a correct formal theory for the kind of composition that is identity. One who thinks that CM is the one correct general theory of the one relation of composition (and the attendant relation of part to whole), as David Lewis did, should, according to me, think that (all) composition is identity. One who merely thinks that there is a part/whole relation that behaves as classical mereology requires, should accept that there is a corresponding kind of composition, and that kind of composition is identity. One may then hold that there are other kinds of composition and parthood: e.g., the relation between a cat and its tail.

23 The compositional nihilist also might chime in, perhaps with some qualifications. The compositional nihilist holds that it never happens that some things compose something else. (Peter van Inwagen entertains the position, under the name 'Nihilism', but does not advocate it, in Material Beings. Rosen and Dorr argue that it is a live possibility in [10].) Now, it depends how this is meant, but it might be that the nihilist will allow that there be a thing that is identical with some things (collectively). If he allows that the atoms (the ones that “make up” a human being) collectively have the property of singing, or being human-shaped, or being human, then why not allow that there is something (but not something distinct from them!) that has these properties? If it can be shown that talk like this does not carry any objectionable further ontological commitment, then why not allow it? (As we will explain in section [6.3], given certain assumptions, the nihilist will be able systematically to “translate” such talk into talk in which quantifiers range only over mereological atoms. This translation can be taken to “cash in” the apparent ontological commitment to things other than atoms.)

24 I believe that there is a special kind of part/whole relation—call it “organic part”—that holds between a cat and its tail, or between a cat and a water molecule that it has ingested, but not between a cat and a steel ball that it has swallowed, nor between a cat and some shrapnel lodged in it. Organic part is not the same as the formal part/whole relation of classical mereology. There are well-know arguments against the claim that a cat is a mere mereological fusion of its parts, turning on temporal and modal considerations—living things grow and diminish, and could have been made of different parts. An under-appreciated argument, I believe, is this: there is no explanation of the fact that the water molecule is i-part of the i-fusion of the cat and the water molecule. But there is an explanation (in terms of life-processes) of why the water molecule counts as part of the cat (and why, for example, a steel ball that it has swallowed does not). If this or other such
Here is an example that all might accept: When you play two notes simultaneously on a piano, middle C and the G a fifth above it, you play a harmonic interval, or dyad. (Think *token* not *type.*) There are at least three numerically distinct sounds: the C, the G, and the dyad. If you can hear normally, you can hear these three sounds. If your hearing is abnormal, you might only be able to hear the C, and not be able to hear the other two sounds. But it is impossible to hear the dyad without hearing the two notes, and impossible to hear the two notes and yet not hear the dyad. (By “hear” I do not mean “consciously discern”.) Why is the one thing possible while the others are impossible? Answer: because the two notes are the dyad. More precisely, the C and the G (collectively) are identical with the dyad. And, therefore, to hear the two notes is to hear the dyad.

### 4.2 Mereological notions and “how many” questions

We may say that identity is at least one kind of composition, if not the only kind. We define its kind of composition in terms of identity: for any things $xx$ and any thing $x$, $xx$ *identity-compose* $x$ just in case $x$ is identical with $xx$ collectively. (Identity-composition is just identity, but we introduce the separate term for suggestiveness and in such a way that a collective reading of the plural is forced, so that we may drop the parenthetical ‘collectively’.) Given the axiom of Comprehensive Collective Identity, we have Universalism for this kind of composition: for any things at all, there is something they identity-compose.

The various mereological notions (relative to identity-composition) are definable in terms of identity. The associated notion of parthood is this: a thing $x$ is an *identity-part* of a thing $y$ (*i-part* for short) if

$$x \text{ and } y \text{ are (collectively) identical with } y$$

Accordingly, say that a thing $x$ is an *i-part* of some things $xx$ (collectively) if

$$(x \text{ and } xx) \text{ (collectively) are identical with (collectively) } xx$$

Arguments are correct, the notion of part answering to formal mereology is a different notion from the notion of organic part.
We may use our new terms to restate the principle that we used Absorption and Symmetry to prove, as follows: if \( y \) is an i-part of \( x \), then \( x \) and \( y \) i-compose \( x \).

Let us say that something \( x \) is an i-atom if

for any things \( xx \), if \( xx \) are (collectively) identical with \( x \), then

every one of \( xx \) is identical with \( x \).

If \( x \) is an i-atom, then \( xx \) are identical with \( x \) only when ‘\( xx \)’ is semantically singular (degenerately semantically plural), as is ‘Hesperus and Phosphorus’. Something is an i-atom if and only if it is its only i-part.

The compositional monist says that there is only one kind of composition, and, hence identity is the only kind of composition, and the definitions above are definitions of the notions of part and atom. The compositional pluralist suggests that there is at least one other kind of composition. The difference matters significantly in our presentation. For suppose we take for granted many ordinary judgments as to what is part of what, for example, that John’s arm is (proper) part of John. If so, then, if compositional monism is right, John is identical with John and his arm (collectively), and yet John is not identical with his arm, so John is not an i-atom. Then, it should follow that there are some things, no one of which is John, such that John is identical with them collectively. On the other hand, if we are pluralists, then perhaps John is an i-atom, i-composed of nothing (other than, trivially, himself). His arm bears some other part-to-whole relation to John. The difference has a significant impact on how we think of the term ‘John’. For if John is an i-atom, then John has a kind of metaphysical singularity that he otherwise does not have. No matter what, ‘John’ is semantically singular. If John is an i-atom, then this semantic singularity has a metaphysical match; otherwise not.

For most terms, there is a unique number that can be called “the number of times that the term refers”. For singular terms, this number is always one; that is why they are semantically singular. We ascribe this number exactly because: if \( t \) is a singular term, then there is something \( x \) that for any objects \( yy \), \( t \) refers to \( yy \) (collectively) if and only if \( yy \) i-compose \( x \). (We assume CCI here.) (Our formulation is meant to entail singular “oneness” as a limit case: there is an \( x \) such that for all \( y \), \( t \) refers to \( y \) if and only if \( y =

\[25\] Strictly speaking, we have not said why that follows, but it is plausible that it is an instance of a general truth, a “remainder principle” about collective identity.
For a typical plural term there is also a uniquely determined number of “times” it refers. E.g., a term t refers twice just in case there are things, x and y, x \neq y, such that: for any things zz, (t refers to zz collectively if and only if (zz i-compose x or zz i-compose y)). An analogous definition would be given for thrice: there are things x, y, and z such that x \neq y, y \neq z, and x \neq z such that.

Thus, a typical referring term, whether singular or plural is associated with a unique referential number. We have seen a way to make this precise when a term refers finitely many times, but we should be able to extend the notion in question for any term that does not refer “too many” times. Some terms, e.g., ‘the sets’ may refer so many times that it does not make sense to speak of their referential number. But otherwise we may say: the referential number of a term t is the cardinality of set S such that for every thing x, x \in S iff t refers to x. Finite referential number is reflected in the object language in a straightforward way: (\exists n z z is one of t) is true, where \exists n z . . . abbreviates the standard first-order formulation of “there are exactly n things z such that . . . ”.

When a singular term refers to an i-atom x, the number one also uniquely characterizes “what the term refers to”—in this case, the object x. (Review the definition of i-atom if it is not obvious why the number one is uniquely associated with x.) Now suppose that a plural term t refers to each of finitely many i-atoms (and to nothing else). Then there is an intuitive match between the referential number of t and the atomic number, as it were, of “what it refers to.”

We may try to define metaphysical numbers for terms by first defining metaphysical numbers for objects, but we run into some trouble if we seek a fully general definition. Roughly, the metaphysical number for an object x is the number of i-atoms xx such that x is identical with xx collectively. This explanation uncritically uses the notion of number in the definiens. What is meant is made more perspicuous by this refinement: let S be the set of i-atoms such that: x is identical with the members of S (collectively), if such a set exists. Then the metaphysical number of x should be defined as the cardinality of S. (Technically, our assumption here of the uniqueness of S requires a further axiom about collective identity; it is “Filtration,” as discussed in section 6.1.) But what if x is i-composed of so many i-atoms that they are “too many” to form a set? Consider the thing x that is identical with all the sets (collectively). (Such a thing exists, by CCI. Paradox looms. See section 6.2.) It is hard to know what “metaphysical
number” to assign to $x$. Worse, there is the problem of things that are not i-atomic. A thing is i-atomic if there are some atoms such that it is identical with them collectively. But there may be things that are not i-atomic; this is the case for a thing that is i-composed of infinitely i-divisible stuff. What metaphysical number should such things be given? What metaphysical number should be given to something that is i-composed of three i-atoms together with some i-atomless stuff? These difficulties for “metaphysical numbers” may be soluble, but we will not investigate further here.

To the extent that the notion of metaphysical number for objects is definable, we might use it to define metaphysical number for expressions. The metaphysical number of a singular term would just be the metaphysical number of the object it refers to. But we face a choice when we come to plural terms. Suppose that Tom and Jerry are i-atoms. Then Genie, their fusion, has metaphysical number two. What metaphysical number should we assign to ‘Tom and Genie’? Three, if we sum the metaphysical numbers of its referents. Two, if we take the metaphysical number of the thing its referents i-compose. Either way, the referential and metaphysical numbers for a term $t$ will be identical if: every thing that $t$ refers to is an i-atom. The second choice is perhaps more natural generally. If we make it, then the metaphysical number for a term may be quite different (either greater or less) than its referential number. For example, ‘Tom and Jerry and Genie’ has referential number three, but has metaphysical number two, while ‘Genie’ has referential number one and metaphysical number two.

5 Semantics

We now outline the basic elements for a semantic theory for a language with plurals. The semantic theory will mesh with our previous metaphysical and semantic considerations.

We will be talking about the “reference” of singular and of plural terms in our object-language, and we will collapse into this notion the idea of “reference” for variables, relative to an assignment.

The crux of the theory is that singular terms refer only once, and plural terms refer multiple times. A plural term $t$ may refer to more than one thing, in the strong sense that it may refer to a thing $x$ and to a thing $y$, with $x$ not identical with $y$. A singular term cannot refer to more than one thing in this strong sense. Each plural term must refer to at least one thing,
but there is no limit to how many things it may refer to.

We now consider the semantic clauses for the crucial forms, ‘identical with’ and ‘is one of’. First, the clause for ‘is one of’ will be:

A sentence of the form

\[ s \text{ is one of } t \]

is satisfied just in case the thing referred to by \( s \) is referred to by \( t \).

Note that if \( s \) refers to \( \alpha \), and \( \alpha \) is identical with (collectively) \( \beta \) and \( \gamma \), then \( s \) refers to (collectively) \( \beta \) and \( \gamma \). This does not cause any trouble for the description ‘the thing referred to by \( s \)’. \( s \) does not refer to \( \beta \), nor to \( \gamma \); only to \( \alpha \). The description ‘The things referred to by \( s \)’ is ambiguous. If it is to be understood as ‘the things collectively referred to by \( s \)’, then it may be infelicitously non-uniquely “satisfied”. (Compare an example of McKay: ‘The students meeting together in the library’ may fail of “uniqueness” if there are three students meeting together in one room of the library, and three (other) students meeting together in some other room in the library.\(^{26}\) On the other reading, expressed unambiguously by ‘the things each of which is referred to by \( s \)’, uniquess will always be met. If \( s \) is a singular term, it is met in the way at the limit: there is just one such thing. In our example, this is the case; \( \alpha \) is the only such thing. When I use the description ‘the things referred to by \( s \)’ (as in the next paragraph) I will always intend this latter interpretation, and uniqueness will always be met.

The clause for the collectively plural identity statements is

A sentence of the form

\[ s \text{ are (collectively) identical with (collectively) } t \]

\(^{26}\) (See Thomas McKay’s \[7\] for a careful discussion of the way ‘the’ behaves in conjunction with plural noun phrases.) Note that if we regard the description as picking out all six students, we get the unhappy result that it is true that

The students who are meeting together in the library are not meeting together in the library.

It’s better to regard the description as infelicitous. It’s better to regard the displayed sentence as semantically problematic (in this circumstance), and truth-value-less or false, rather than as straightforwardly true.
is satisfied just in case the things referred to by \(s\) (collectively) are identical with (collectively) the things referred to by \(t\).

Now suppose, for illustration, that someone simultaneously plays the notes C, E, and G on the piano. We take it that

- ‘the C’ refers to \(c\), that token of the note C
- ‘the E’ refers to \(e\), that token of the note E
- ‘the G’ refers to \(g\), that token of the note G
- ‘the C and the G’ refers to \(c\) and refers to \(g\)
- ‘the CG dyad’ refers to \(x\), the thing identical with (collectively) \(c\) and \(g\)
- ‘the EG dyad’ refers to \(y\), the thing identical with (collectively) \(e\) and \(g\)
- ‘the triad’ refers to \(z\), the thing identical with (collectively) \(c\), \(e\), and \(g\).

Note that, ‘the CG dyad’ does not refer to \(c\), and that ‘the C and the G’ does not refer to \(x\).

Intuitively, the following are true:

- The C is one of the C and the G.
- It is not the case that the C is one of the CG dyad.
- The C and the G (collectively) are identical with the CG dyad.
- The C, the E, and the G (collectively) are identical with (collectively) the CE dyad and the CG dyad.

Our semantic clauses ensure this is the case.

Turning to one-place predicates generally, we simply interpret them with properties, so that a sentence of the form

\[s \text{ is } F\]

(with \(s\) a singular term) is satisfied just in case the thing referred to by \(s\) has the property expressed by the predicate \(F\).

If \(t\) is a plural term, then

---

27It is perfectly possible to give a more “extensional” semantics for predicates, on which we do not appeal directly to properties. We would take predicates to “refer” to objects, and give appropriate clauses for satisfaction. One-place predicates would have a reference relation formally the same as that for plural terms: they would refer many times over, each time to a thing (or, equivalently, to some things collectively). Two-place
t are collectively $F$

will be satisfied just in case the things referred to by $t$ collectively have
the property expressed by $F$. Two-place predicates are interpreted by two-
place relations, and so forth for predicates and properties of higher arity.

Formalizing distributive plural predication could be achieved by forc-
ing the original sentence into a form so that ‘is one of’ is the only distribu-
tive predicate (as it is in some formalisms of plurals). Then the clauses for
‘is one of’ and for singular predication would suffice.

To keep things simple, let that be our official policy. It is worth noting,
if only in this paragraph, that we could allow distributive features with a
more natural surface syntax. For example,

each of $t$ is $F$;

(with plural term $t$) is satisfied just in case: each thing that $t$ refers to satis-
fies ‘is $F$’. Similarly,

one of $t$ is $F$;

(with plural term $t$) is satisfied just in case: some thing that $t$ refers to sat-
ifies ‘is $F$’. We could allow analytically distributive predicates, analogous
to plural common nouns in English. We would have a syntactic operation
(add an ‘s’) and a corresponding semantic clause so that, e.g.,

t are $Fs$

would be satisfied just in case every thing that $t$ refers to satisfies $F$.

Given our official policy, however, it follows from our semantics that
for any plural terms $tt$ and $ss$

$tt = ss \rightarrow (\phi(tt) \leftrightarrow \phi(ss))$

predicates would require a three-place reference relation, relating the predicate many
times over, each time to a thing (or some things collectively) and a thing (or some things
collectively) in order. Though this kind of semantics does not interpret predicates with
properties, it requires some contentful assumptions about the availability and plenitude
of these reference relations.

Yet another semantics could be given, using sets in the interpretation of predicates. But
the most natural ways to do this raise difficulties about the interaction of set theory with
collective identities, discussed below.
How Composition Could be Identity

(where ‘=’ expresses collective identity) is valid provided that tt does not occur as an argument of ‘is one of’ (on the right), in \( \phi \). This corresponds with (indeed, follows from our semantics together with) the thesis that identicals share all their properties. This is the restricted form of substitutivity that we should expect.

The more exacting condition of “distributive identity” guarantees full substitutivity of two plural terms.

\[
\forall x (\text{OneOf}(x, \text{tt}) \leftrightarrow \text{OneOf}(y, \text{ss})) \rightarrow (\phi(\text{tt}) \leftrightarrow \phi(\text{ss}))
\]

is valid, for any \( \phi \). This unrestricted substitutivity follows from the fact that the truth of a “distributive identity” sentence requires that the two plural terms involved be semantically identical: any thing the one refers to, the other refers to, and vice-versa; and there is nothing more to these terms, semantically.

6 Glimpses Beyond

6.1 NTI, Restricted NTI and mereological notions

We have just seen that

\[
\forall x (\text{OneOf}(x, \text{tt}) \leftrightarrow \text{OneOf}(y, \text{ss})) \rightarrow (\phi(\text{tt}) \leftrightarrow \phi(\text{ss}))
\]

is valid in the semi-formal semantics. We get full substitutivity for “distributively identicals” and only partial substitutivity for “collectively identicals.” This is the failure of an interesting principle, that we may call the Numerical Transparency of Identity (NTI). NTI says that any construction of the form

\( (\text{collectively}) \text{ are identical with } \) (collectively)

is logically equivalent with the corresponding instance of

for all things, that thing is one of (collectively) if and only if it is one of (collectively)

where the substituends for (collectively) and (collectively) are appropriate. Given our notions of referential and metaphysical number, NTI, intuitively, says that referential number is metaphysical number.

There is a restricted version of NTI that is true. Here is a material statement of it: let (material) Restricted NTI (RNTI) be the claim

\( (\text{material}) \text{ (collectively) are identical with } \) (collectively)
For any i-atoms \( xx \) and any i-atoms \( yy \):
\[
xx \text{ are (collectively) identical with (collectively) } yy
\]
if and only if
\[
\text{for any thing, it is one of } xx \text{ if and only if it is one of } yy.
\]

RNTI follows from Absorption and Symmetry, with one new axiom. The new axiom is best stated with the help of one more definition. Say that a thing \( x \) and a thing \( y \) i-overlap if

there is something \( z \) such that \( z \) is an i-part of \( x \) and \( z \) is an i-part of \( y \)

The new axiom, \textbf{Filtration} is

For any things \( xx \), and any things \( yy \)
\[
\text{if } xx \text{ (collectively) are identical with } yy \text{ (collectively)}
\]
then every one of \( xx \) i-overlaps one of \( yy \).

It is therefore plausible that there is a restricted version of Unrestricted Substitutivity of Collectively Identicals (USCI) that holds:

\textbf{If}
\[
\text{\( \varphi(\overline{1}) \iff \varphi(\overline{2}) \) for any (non-traditionally-opaque) context } \varphi. \text{ For}
\]
\[
\text{for all things, that thing is one of } \overline{1} \text{ if and only if it is one of } \overline{2}
\]
\textbf{then}

should guarantee the substitutivity.
6.2 For any things, a thing

Recall the axiom of Comprehensive Collective Identity (CCI).

For any things, they are collectively identical with some thing.

CCI raises some technical issues. It tells us that there is a “function” from the thingses, so to speak, to the things. And since it is identity, it is a “one-one function”: for any things \( x_{xx} \) and any things \( y_{yy} \), and any thing \( x \), if they \( x_{xx} \) (collectively) are \( x \) and they \( y_{yy} \) (collectively) are \( x \), then they \( x_{xx} \) (collectively) are (collectively) they \( y_{yy} \). Does this violate Cantor’s theorem? No. CCI does not require that there be a one-one function from the sets of things to the things.

Let \( xx \) be some things that are closed under i-composition: for any things \( yy \), if every one of \( yy \) is one of \( xx \), then there is something \( z \) such that \( yy \) i-compose \( z \) and \( z \) is one of \( xx \). (Being closed under i-composition is not a property of some things, since it is defined in terms of ‘is one of’. But there is no reason to deny that it is a property of the set of those of things, if it exists.) Suppose \( xx \) “form a set;” that is, there is a set \( S \) such that for all things \( x \), \( x \in S \) iff \( x \) is one of \( xx \). Cantor’s theorem tells us that there are strictly more subsets of \( S \) than there are members of \( S \). Say that two sets are “i-equivalent” if the members of the one are collectively identical with the members of the other. I-equivalence is an equivalence relation. CCI tells us, in effect, that for any non-empty subset \( T \) of \( S \), there is a unique \( x \in S \) such that \( \{ x \} \) is i-equivalent with \( T \). Let \( f \) be the function such that for any non-empty \( T \subseteq S \), \( f(T) \) is the relevant \( x \in S \). Cantor’s theorem requires that there are many cases of \( T, U \) subsets of \( S \) such that \( T \neq U \) while \( f(T) = f(U) \). Absorption and Symmetry entail that there will be many cases like this, so we should be alright. These are basically cases in which NTI fails, put in terms of set-membership instead of “one of”: some member of \( T \) is not a member of \( U \) (or vice-versa), yet the members of \( T \) (collectively) are identical with the members of \( U \) (collectively).

There are genuine technical difficulties, however, involving set theory. The expression “the set that contains the things \( xx \)” is ambiguous; it could mean “the set whose members are every one of \( xx \) (and no other things)”. But it could mean “the set that bears the membership relation to \( xx \) collectively (and to no other things).” The former is what we would usually...

28 But only so to speak: as explained in section 2, I do not believe there is any point, metaphysically, in plurally plural quantification.
mean: e.g., the set of dogs has each dog as a member, and has no other members. The set picked out by the latter, however, is a singleton, in the sense that it bears the membership relation *once and only once*, if it exists. It is the singleton of the thing $y$ such that $xx$ collectively are $y$, (if such a thing $y$ exists).

CCI says that for any things $xx$, such a $y$ does exist. But now we confront issues with some formal resemblance to the cardinality issue above, and connected with Russell’s paradox. In fact, if every singleton is an i-atom, then it takes only the most basic further assumptions to get a contradiction.\(^{29}\)

This might make us doubt CCI.

First, it is worth noting that the same issue besets the axioms of Classical Mereology (CM), for in CM we have the very similar axiom that for any objects, there is a fusion of them (that is a single object). If every singleton is a mereological atom, then, with a fairly weak set theory, a contradiction ensues.\(^{30}\)

Second, and more importantly, we are far from forced into a contradiction. It is not clear what the best response is, however. Apparently, we should say that not every singleton is an i-atom. Perhaps there are unexpected relationships among sets whose members are not i-atoms. At any rate, it appears that a safe attitude is this: set theory was formulated with i-atoms in mind, in singular logic. If we restrict its formulation in our system so that something is a member only if it is an i-atom, all should be fine. Pure set theory will not be affected, if we suppose, as is plausible, that all pure sets are i-atoms. Only impure set theory requires reconsideration. Fortunately, it would be basically unaffected if the only i-complexes that are allowed to be members of sets are *ur-i-complexes*: i-complexes that have

---

\(^{29}\)Say that a singleton is “naughty” if there are some singletons that collectively bear the membership relation to it, and it is not among them. The following comprehension scheme is fundamental to the logic of plurals:

$$\exists x \phi(x) \rightarrow \exists x \forall x (\phi(x) \leftrightarrow \text{OneOf}(x, xx))$$

If there is more than one singleton, then there is a naughty singleton. Let $\phi(x)$ be “$x$ is a naughty singleton” and a contradiction ensues. It is crucial to the derivation of the contradiction that Material NTI (with quantifiers restricted to the singletons) applies.

\(^{30}\)Gabriel Uzquiano makes essentially the same point, and further points about the difficulties of combining unrestricted mereology, set theory, and plurals in [*14*]. Uzquiano is skeptical that the two full theories can be satisfactorily combined in full generality.
no sets as i-parts. We may assume that concrete objects never have sets as i-parts, and so there should be no problem with the standard hierarchy of impure sets “built up” from a base of concrete things.

One last technical matter should be addressed, connecting i-composition with Classical Mereology (CM). If we add another axiom, the analog of what is called, in CM, the Strong Supplementation Principle (SSP), then we can tell that the i-part relation behaves exactly as CM says the ‘part’ relation behaves. SSP says:

For any thing $x$, and any thing $y$ if all i-parts of $x$ i-overlap $y$, then $x$ is an i-part of $y$.

In the resulting theory, to be a mereological atom is to be an i-atom; for something to be the mereological fusion of some things is for them to i-compose it, and so forth. SSP has some intuitive plausibility, but it does not seem to be of quite the same character as CCI, Absorption, Symmetry, and Filtration.

### 6.3 I-atoms and RNTI

RNTI tells us that NTI holds for a restricted “range of things”: it holds of i-atoms. Thus, if our plural (and singular) quantifiers were restricted in such a way as to range only over i-atoms, then NTI would hold within the restricted language; hence, so would USCI. Would we be missing any expressive power? Perhaps not, if all things are (identical with some) i-atoms. If so, we will show, then it is possible to give a semantics for a language in which NTI fails, in a meta-language in which NTI holds (and hence, plausibly, USCI; we will focus on NTI).

Suppose that

Everything is identical with (collectively) some i-atoms—that is: for any thing $x$, there are some i-atoms $xx$ such that $xx$ are (collectively) identical with $x$.  

---

31Here is one way to formulate CM, using plural quantification. (This formulation traces back to Tarski [12].) Adjust your notion of “part”, if necessary, so that everything is part of itself. Say that two things ‘overlap’ if they have a common part. Say that a thing $x$ is the ‘fusion of’ some things if: every one of them is part of $x$ and every part of $x$ overlaps one of them. We can now state the axioms as:

(1) The part-relation is transitive;
(2) For any things, there is a unique fusion of them.

---
Then for any thing, there are some i-atoms that are (collectively) it.

We can exploit this fact, given RNTI, to give a semantics for a plural language, using a meta-language in which we quantify only over i-atoms. If we restrict the quantifiers of our meta-language to i-atoms, we can still say everything we could say without the restriction, with a little ingenuity. The effect of this restriction is that the sentences

For any thing, it is an i-atom.
For any things, each of them is an i-atom.

become true in our meta-language. Where, before, we had a thing identical with some i-atoms (collectively), we now just have those i-atoms. But any property had by the old thing is had (collectively) by the (new) i-atoms (since the old thing was identical with them). So, essentially, nothing has been lost; we can still effectively refer to the thing, by referring to the i-atoms that are (collectively) it. (Given the uncontroversial RNTI), in our restricted language, Material NTI holds.

The technique applied to an example

Consider again the example in which three notes are played on the piano, supposing that each note is a i-atom. Consider the CG dyad, \( x \). Where, before, we would have said, for example

\( x \) satisfies ‘is a harmonic fifth’

we replace this with

\( c \) and \( g \) (collectively) satisfy ‘is a harmonic fifth’

Similarly, in our semantic account, we will not say that

the CG dyad

refers to \( x \) (the CG dyad). We will say that it refers to (collectively) \( c \) and \( g \).

In general, where we would have said a singular term refers to something, we now say it refers to (collectively) some i-atoms (\textit{sotto voce}: the ones that i-compose that thing). We will say that predication of a predicate \( F \) of a singular term \( t \) results in truth (or satisfaction) when the things (collectively) referred to by \( t \) (collectively) satisfy \( F \).

Things get trickier when we consider plural terms that refer to more than one i-complex. Where, before, we would have said that
Each of $x$ and $y$ satisfies ‘is a consonant interval’
we now must say

$c$ and $g$ (collectively) satisfy ‘is a consonant interval’; and
$c$ and $e$ (collectively) satisfy ‘is a consonant interval’.

Where, before, we might have said

$x$ and $y$ (collectively) satisfy ‘is a major triad’
we now must say

$c$, $e$, and $g$ (collectively) satisfy ‘is a major triad’.

Similarly, in our semantic account, we will not say that

the CG dyad and the CE dyad

refers to $x$ and refers to $y$. Instead, we will say that it refers to $c$ and $g$
(collectively) and refers to $c$ and $e$ (collectively). It does not refer to $c$, nor
to $e$, nor to any i-atom; nor does it refer to $e$ and $g$ (collectively), nor to $c$, $e$,
and $g$ (collectively).

When we interpret

each of the CG dyad and the CE dyad is a consonant interval
we calculate it as true via this calculation: for any i-atoms that ‘the CG
dyad and the CE dyad’ refers to (collectively), they (collectively) satisfy ‘is
a consonant interval’.

But when we interpret

the CG dyad and the CE dyad (collectively) are a major triad
we calculate it as true because: $c$, $e$, and $g$ (collectively) satisfy ‘is/are a
major triad’. But how did we get those three out of the reference of the
plural term?
The key to interpreting collective predication

The general principle in effect here is articulated, as applied to ‘the CG dyad and the CE dyad’:

The i-atoms $xx$ such that: (1) every one of $xx$ is one of some i-atoms that (collectively) ‘the CG dyad and the CE dyad’ refers to; and (2) any i-atom that is one of some i-atoms that (collectively) ‘the CG dyad and the CE dyad’ refers to is one of $xx$—those i-atoms, $xx$, collectively satisfy ‘is/are a major triad’.

Note that we cannot simplify this. The i-atoms $xx$ cannot be called “the i-atoms that the plural term refers to”. The plural term does not refer to any one of $c, e$, and $g$! That definite description is infelicitous (and empty).

The trickiest part of all involves the interpretation of plural quantifiers. We need an appropriate notion of the semantic value(s) of a plural variable relative to an assignment. We think of the assignment as a relation, that relates variables to things in a very flexible way: it can relate the variable to a single i-atom, or it can relate the variable to (collectively) many i-atoms; and it can do this more than once. For example, it may relate a variable to (collectively) i-atoms $\alpha$ and $\beta$, and also to (collectively) i-atoms $\beta$ and $\gamma$, without relating the variable to $\alpha$ or to (collectively) $\alpha$ and $\gamma$.

One-place predicates will get a similar kind of interpretation, but with the possibility of emptiness. $n$-place predicates require $n + 1$-place “reference” relations.

You cannot get something from nothing. The expressive power of plural quantifiers in the object-language is captured only by the use of these assignment relations in the meta-language. Accordingly, we have to make serious, though plausible, assumptions, in the meta-language, about the existence of relations. We do not have the room to pursue the details here. (See the appendix.)

The reduction of i-complexes

We have seen that if everything is identical with some i-atoms, then we can give a semantics in which we quantify only over i-atoms.

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$^{32}$Cf. footnote $^{26}$
This fact can be taken in different ways. It might simply be taken as an interesting “theorem” that follows from the logic of plurals and identity. But it might be taken to show that the object-language, in which Material NTI fails, ought to be thought of as some kind of code for the meta-language, in which Material NTI holds. Further, it might be said that, in principle, such a language exists, and is superior: it gives a more transparent representation of the world. In a transparent language, singular terms refer to metaphysically singular things, and plural terms refer to a unique number of these things—in the terminology of section 3.2 referential number matches metaphysical number. Languages in which Material NTI fails feature a kind of “numerical opacity” in their terms and quantifiers. It might even be urged that a proper understanding of a numerically opaque language translates it into a numerically transparent language—one in which NTI holds.

This is perhaps the attitude that the compositional nihilist takes. In the best language, our terms and quantifiers range only over mereological atoms (i-atoms). In this language, it is true to say “there are no composite objects”. Such a language exists, in principle. But ordinary and even scientific language is not like this. In ordinary and non-fundamental scientific language, there appears to be reference to composite objects. Classical Atomistic Mereology will come out logically valid, given the semantics or interpretive technique that we have suggested. That technique, however, allows us to systematically translate the non-fundamental language into the “superior” language in which NTI holds, and in which there is not even apparent quantification over anything but atoms.

It is not implausible that the required restriction on the quantifiers is not ontologically significant. This is the other side of the ontological innocence of i-composition: talk about them as many, never as one—you can still say everything there is to say. Whatever the merits of this attitude, it requires i-atomism: everything is identical with some i-atoms. Should this be false, there simply is no general escape to a language in which NTI holds.

References


A Appendix: Formalization

We will now present a full formal system, with semantics, in which (the formal analog of) Material NTI fails, but we give a semantics in a metalanguage in which Material NTI holds. Syntactically, the language is an expansion of a standard first-order language, expanded to include plural terms and quantifiers.

We have argued that ‘one of’ and ‘each of’ are modes of predication, rather than relations. Thus, they appear to combine with a plural term (“on the right”) to form a quantifier. Thus it would be appropriate to formalize them so that, for example “Each of xx is F” looks like $\forall(x, xx) F(x)$ and “x is one of xx” looks like $\exists(y, xx) x = y$. A reasonable logic (in the sense of rules) for the resulting quantifiers is interestingly similar to the rules for quantifiers in free logic. For example, $\forall(x, xx) \phi(x)$ does not yield $\phi(a)$ for arbitrary $a$; but it does in conjunction with $\exists(x, xx) x = a$.

Here, we will consider instead a more mainstream syntax, in which “is one of” is treated more like a unit, and is to be used to capture the meaning of any use of ‘one of’ or ‘each of’. It is easy to see that our semantics can be adjusted for the more appropriate syntax above, but we present the more mainstream version so that it will be clearer, to those used to that syntax, just how the semantics works.

The language

\[
\begin{align*}
\text{Terms:} & \quad \begin{aligned}
\text{Singular terms:} & \quad \{ \text{Names: } a, b, c, \ldots \\
& \quad \text{Variables: } x, y, z, \ldots \\
\text{Plural terms:} & \quad \{ \text{Names: } aa, bb, cc, \ldots \\
& \quad \text{Variables: } xx, yy, zz, \ldots \\
& \quad \text{Lists: } a + x, a + x + xx, \ldots
\end{aligned} \\
\text{Quantifiers: } & \quad \exists, \forall \text{ (can bind both kinds of variables)}
\end{align*}
\]
How Composition Could be Identity

Predicates:

\[
\begin{align*}
\text{Non-logical:} & \quad F, G, R \ldots \text{ (each with a fixed arity, } n, \\
& \quad \text{saturated by any } n \text{ terms to form an atomic wff)} \\
\text{Logical:} & \quad = \text{ (2-place)} \\
\text{Pseudo:} & \quad [tt] \text{ (1 singular place), for every singular or plural term } tt
\end{align*}
\]

Propositional connectives: \( \neg, \land, \lor, \rightarrow, \leftrightarrow \)

The syntax is as you would expect, and we do not give a precise statement of it. Pseudo-predicates have the same syntax as normal one-place predicates except that they only accept singular terms as arguments. For example, the one made from \( xx \) expresses ‘is one of \( xx \)’.\(^{33}\) Otherwise, plural and singular terms can go in all of the same places.

Here are some examples. Sentences in the same box are logically equivalent.

<table>
<thead>
<tr>
<th>John and Paul each outweigh Ringo.</th>
<th>( R(a, c) \land R(b, c) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \forall x(<a href="x">a + b</a> \rightarrow R(x, c)) )</td>
</tr>
<tr>
<td>John and Paul collectively outweigh Ringo.</td>
<td>( R(a + b, c) )</td>
</tr>
<tr>
<td>Some thing outweighs Ringo.</td>
<td>( \exists x R(x, c) )</td>
</tr>
<tr>
<td>Some things (at least two) out weigh Ringo.</td>
<td>( \exists xx (\exists x \exists y (<a href="x">xx</a> \land <a href="y">xx</a>) \land x \neq y \land R(xx, c)) )</td>
</tr>
<tr>
<td>John and Paul are collectively identical with (collectively) ( xx )</td>
<td>( a + b = xx )</td>
</tr>
<tr>
<td>Each of John and Paul is one of ( xx )</td>
<td>( \forall x([a + b]x \rightarrow [xx]x) )</td>
</tr>
</tbody>
</table>

Note that when a plural term occurs directly as an argument of a pred-

\(^{33}\)We could let them accept plural arguments. We would have a choice of two natural interpretations of the result: (e.g.,) either as “are among \( xx \)”, or as “are (collectively) identical with some thing that is one of \( xx \)”. 

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icate, including the identity predicate, we always interpret the predication as collective. Thus there is no distributive/collective ambiguity in our language. Distributive plural predication is represented through a surrogate that functions in many ways like ‘is one of’ functions on the Standard Line: the pseudo-predicates formed by putting brackets around a plural term. (We do not interpret these as if they expressed the holding of a simple relation between the term in brackets and the other term, of course; rather they are interpreted as disjunctively distributing the identity predicate, in accord with section [3].)

**Formal Semantics**

First, we assume we are speaking a meta-language in which the quantifiers are restricted to i-atoms.

A model will consist of a domain \( D \) of objects, and a battery of *interpretation relations* appropriate to \( D \). There is no need to think of \( D \) as a set; indeed, we may think of ‘\( D \)’ as a plural term, so that \( D \) are some i-atoms.

To get our full formal semantics, there must be enough relations so that the notion of arbitrary interpretation relations and an arbitrary variable assignment, for the domain \( D \), do their intended work. We will make some strong, but plausible, assumptions about the plenitude of properties of things among \( D \). (As far as I know, they are consistent, for any domain at all. If they are not, this does not show that there are not enough relations to do the needed work; it only shows that this way of ensuring that will not always work.)

Here are our principles. The first two are analogous to what, in a classical setting, we might put roughly as “there is a property for any arbitrary extension (set of things)”. The second two link properties to relations in such a way as to guarantee that there will be analogs of the classical notion of an arbitrary variable-assignment. Consider the objectual quantifiers to be restricted to \( D \).

1. For any things, there is a property had by them collectively, and had by nothing else.

2. For any properties, there is a property that is had by a thing or some things (collectively) if and only if it or they have one of those properties.
3. For any things \( t_1 \) and any property, there is a (two-place) relation such that (A) things \( t_1 \) (collectively) bear it to some things \( t_2 \) (collectively) just in case they \( t_2 \) (collectively) have the property; and (B) the relation relates no other things.

4. For any (two-place) relations, there is a (two-place) relation that relates some things (collectively) to some things (collectively) just in one of those relations does.

A model consists of a domain \( D \), some objects, and some interpretation relations \( I_1, I_2, \ldots \) for the names and predicates. \( I_1 \) is for singular and plural names and for one-place predicates; \( I_n \) for \( n \geq 2 \) is for \( n \)-place predicates.

The interpretation relations relate names and predicates appropriately to objects from (among) \( D \). \( I_1 \) has an arity appropriate for names and one-place predicates—it is a two-place relation. In general, \( I_n \) is an \( n + 1 \)-place relation, relating predicates of arity \( n \) (where terms have arity 1) to objects taken in an order. In particular:

For each singular constant term \( \lceil a \rceil \), there are some thing(s) \( t_1 \) \(^{34}\) from \( D \) such that:

- \( a \) bears \( I_1 \) to (collectively) \( t_1 \); and
- \( a \) does not bear \( I_1 \) to any other things.

For each 1-place predicate \( F \) and plural constant term \( tt \), either it bears \( I_1 \) to no things, or it bears \( I_1 \) to some thing or things (collectively) (and possibly some other things). Any things (collectively) that it bears \( I_1 \) to are such that each of them is one of \( D \).

For each 2-place predicate \( R \), either the predicate bears \( I_2 \) to no objects, or there are some things \( t_1 \) (among \( D \)) and some things \( t_2 \) (among \( D \)) such that \( R \) bears \( I_2 \) to \( t_1 \) (collectively) and \( t_2 \) (collectively) (taken in order). \( R \) may bear \( I_2 \) similarly, arbitrarily many times (i.e., to (some other things and other things) and to (some still other things and still other things), etc.)

\( I_n \) works similarly for \( n \)-place predicates.

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\(^{34}\)Here and throughout, when we say “there are some things”, we mean to include the possibility that there is just one.
Truth at a model

To define truth at a model, we will need the notion of an assignment relation for domain $D$. An assignment relation $A$ for domain $D$ will relate each singular variable to some things (collectively), and will relate each plural variable to some things (collectively), and possibly to more things (collectively), just as an interpretation relates singular names and plural names. For convenience, we will extend the notion of an assignment relation so that all terms get reference on the relation: Officially, an assignment relation in a given model must also relate each name in exactly the way that the interpretation relation relates the names to members of $D$.

Further, for any terms $t_1$ and $t_2$, it must relate $t_1 + t_2$ to any things that it relates $t_1$ to and to any things it relates $t_2$ to, and to no other things.\footnote{For full rigor, we stipulate that the semantic value for terms formed with $+$ are calculated from left to right, so that $a + b + c$ is treated by examining $a$ and $b + c$, etc. The opposite order of evaluation would produce the same results.}

For each variable $⌜z⌝$, an assignment relation $B$ is a $z$-variant of an assignment relation $A$ just in case $A$ and $B$ relate all variables, except possibly $z$, identically; that is, for any other variable, if $A$ relates that variable to some things (collectively), then $B$ relates that variable to them (collectively), and vice-versa.

We can now define satisfaction at a model $(D, I)$ relative to an assignment relation $A$ as follows:

atomic wffs

- **pseudo-predicates** (where $t_1$ is plural and $t_2$ is singular) $⌜[t_1](t_2)⌝$ is satisfied relative to $A$ just in case $t_1$ bears $A$ to (collectively) the things that $t_2$ bears $A$ to (collectively).

- **identity** $⌜t_1 = t_2⌝$ is satisfied relative to $A$ just in case any thing that is one of some things that $t_1$ bears $A$ to (collectively) is one of some things that $t_2$ bears $A$ to, and vice-versa.

- **Non-logical predicates** If $F$ is a one-place predicate and $t$ is a term, the atomic wff $⌜Ft⌝$ is satisfied relative to $A$ just in case $F$ bears $I$ to (collectively) the things $things_1$ such that (1) every one of $things_1$ is one of some things that $t$ bears $A$ to (collectively), and
(2) any thing among some things that \( t \) bears \( A \) to (collectively) is one of things_1.
For \( n \)-place predicates, the satisfaction condition for \( \forall F(t_1 \ldots t_n) \) is similar, with things_1 as above for \( t_1 \), and things_2 similarly chosen for \( t_2 \), etc.

**wffs formed by propositional connectives** (treated as usual)

**quantified wffs** For every singular or plural variable \( z \), a wff \( \forall z \phi(z) \) is satisfied relative to \( A \) just in case, for every \( z \)-variant of \( A \), \( \phi(z) \) is satisfied relative to that \( z \)-variant. (Similarly for the existential quantifier.)

Truth at a model for a closed sentence is defined as satisfaction relative to all assignment relations for that model.

### A.1 Results

**Substitution**

As a result, the traditional scheme of substitutivity of identicals will fail, for plural terms, though it will hold for singular terms. A weakened version will hold for plural terms, and unqualified substitutivity holds when plural terms are “hyper-identical”.

(Let us introduce the usual abbreviation for hyper-identity:

\[ tt \approx ss \]

abbreviates

\[ \forall x([tt]x \leftrightarrow [ss]x) \]

where \( x \) is not free in \( tt \) or \( ss \).

Valid versions of substitutivity:

\[ t = s \] where \( t \) and \( s \) are singular terms, and \( \phi(s) \) arises from \( \phi(t) \) by replacing an occurrence of \( t \) in \( \phi(t) \) with \( s \).

\[ \phi(t) \]

\[ \phi(s) \]
tt = ss where tt and ss are plural terms, and φ(ss) arises from
φ(tt) by replacing an occurrence of tt in φ(tt) that is not
within brackets with ss.

tt ≈ ss where tt and ss are plural (or singular) terms, and φ(ss)
φ(tt) arises from φ(tt) by replacing an occurrence of tt in
φ(ss) φ(tt) with ss.

Formal Mereology

Classical Atomistic Mereology is semantically valid, with the following
(abbreviatory) definitions of the mereological relations.

Abbreviations:

≤ (is part of) e.g., x ≤ y abbreviates x + y = y
○ (overlap) x ○ y abbr. ∃z(z ≤ x ∧ z ≤ y)

fu (fusion) fu(x, xx) (x is the fusion of xx) abbr.

∀y([xx]y → y ≤ x) ∧
∀y(y ≤ x → ∃z([xx]z ∧ y ○ z)

at (atom) at(x) abbr. ∀y(y ≤ x → y = x))

The semantically validated axioms then appear as:

Transitivity ∀x∀y∀z((x ≤ y ∧ y ≤ z) → x ≤ z)

Atomism ∀x∃y(at(y) ∧ y ≤ x)

Unique Fusions ∀xx ∃!x fu(x, xx)

36I omit the qualifications needed to avoid “variable collisions”.

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