$\star$ 1. Calculate the Plücker coordinates for the projective closure of the affine line parametrized by

$$t \mapsto (1,4,2,1) + t(0,2,0,5) \in \mathbb{A}^4 \subset \mathbb{P}^4$$

$\star$ 2. As presented in lecture 9, there are 16 Plücker relations for $G(2,4)$. Explicitly list these 16 relations and show that only one of these relations is necessary, i.e., the ideal generated by these is generated by one of the Plücker relations.

$\star$ 3. In lecture 9, we defined the standard open sets $U_{i_1,\dots,i_r} \subset G(r,n)$.

(a) Describe the points $L \in G(r,n)$ that are in $U_{1,\dots,r}$ but in no other standard open set.

(b) Describe the points that $L \in G(r,n)$ that are in $U_{1,\dots,r}$ and exactly one other standard open set.

$\star$ 4. We have seen that the Plücker embedding allows us to consider the Grassmannian of lines in $\mathbb{P}^3$ as a quadric hypersurface $G$ in $\mathbb{P}^5$.

(a) Fix a point $p \in \mathbb{P}^3$ and let $X$ be the set of lines passing through $p$. Show that $X$ is a plane lying on $G$. (Hint: up to a linear change of coordinates, you may assume that $p = (0,0,0,1)$.)

(b) Fix a line $L \in \mathbb{P}^3$ and let $Y$ be the set of lines meeting $L$. Show that $Y$ is the intersection of $G$ with a hyperplane. (Hint: again, up to a change of coordinates, you are free to assume that $L$ is your favorite line.)

5. In $\mathbb{R}^3$, consider the following three lines:

$$L_1 = Z(y,z), \quad L_2 = Z(x-z, y-1), \quad L_3 = Z(z-2x, y-2).$$

(a) The union of all lines $L$ such that $L$ meets each of these three lines forms a surface, $S$, defined by an equation $f = 0$. Find $f$.

(b) Fix a random line $L$. How many lines (most probably) meet $L$, $L_1$, $L_2$, and $L_3$? (Hint: Where does $L$ meet the surface defined by $f = 0$?)

6. In $\mathbb{R}^3$, choose four generic lines $L_1$, $L_2$, $L_3$, and $L_4$ with the condition that $L_1$ and $L_2$ intersect and $L_3$ and $L_4$ intersect. Describe the lines meeting all four of these lines.