Power Laws in the Stock Market

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Table of Contents

List of Figures

Abstract

The purpose of this thesis is to assess the possibility of the existence of a power-law behavior in the stock market. First, it presents recent research that challenges standard finance theory. Second, it shows how herding behavior in the stock market could lead to great volatility. Finally, it develops a model that attempts to explain the herding behavior based on transmission of information in the stock market. This model is an agent-based artificial model based on the sandpile model developed by Bak, Tang, and Wiesenfeld in 1987. It relies on the assumption that communication networks can be represented by graphs with edges and vertices, where each vertex is a trader and an edge denotes the connection between two traders. Experiments using this model show that information sharing is highly dependent on the degree of connectivity among the traders, the rate of information being lost, and the traders' willingness to share information. For some of the networks considered, the distribution of information transmission was found to behave like a power-law.

Introduction

Recent research has challenged standard finance theory, and proposed other ways of analyzing the stock market. Physicists have proposed that the stock market behaves like a self-organized critical system, which can be defined as a system where the relationship between cause and effect is not linear. Bak, Tang, and Wiesenfeld (1987) were the first ones to introduce the concept of self-organized criticality to explain the behavior of their sandpile model. Soon afterwards this theory was used to justify the complexity of natural systems, including the stock market. One of the important features of a self-organized critical system is that it exhibits a power-law behavior, and thus the search for self-organization was replaced by the search for power laws. A series of power laws were found to describe the behavior of stock market returns, trading volume, and herding behavior of traders.

The high volatility in the stock market was explained by the informational events leading to herding behavior. Therefore, the way information is transmitted and network of traders are of extreme importance for analyzing stock market returns.

In this thesis, I investigate the possibility that the stock market behaves as a self-organized critical system that follows a power-law behavior. The organization of this thesis is as follows. The first chapter provides an overview of the standard finance theory and its implications. Chapter 2 describes the theory behind self-organized critical systems, the sandpile model and power laws. Chapter 3 analyzes the phenomenon in the stock market referred to as herding behavior. Three main models of herding behavior are briefly discussed in this chapter. A model of information transmission, strongly related to herding behavior is proposed in Chapter 4 and experiments based on this model are conducted in Chapter 5. The conclusion summarizes the main findings of Chapter 5 and suggests possible directions for future research.

Chapter 1

Basic Concepts of the Financial Market

The stock market has been the center of numerous studies since the beginning of the twentieth century. The majority of these studies have concentrated on one of the most basic characteristics of the stock market—the distribution of the returns.[1] Over the last few years new studies have been challenging the standard finance theory and offering new perspectives on the dynamics of the stock market. It is necessary to be familiar with the standard financial theory in order to understand the implications of these new theories. The following two sections explain the basic terminology of the stock market.

1.1 Financial Assets

A financial asset is an asset that has a certain value because of a legal claim to a future cash flow. Examples of financial assets include stocks, bonds, bank deposits, and so on. Financial assets are commonly classified as debt or equity instruments:

[1] Section 2 will provide different formulas for calculating these returns.

1. Debt instruments include government bonds or bills, corporate bonds, mortgages, and bank loans. The holder of the debt claim has two sources of cash flow: the original value of the claim and an interest on the loan (both received at the end of the contract).

2. The most well-known form of equity instruments is company shares. The holder has three sources of cash flow: dividend payment, capital gain (when the share is sold above its purchase price) and a portion of the value of the institution issuing the shares (when the institution is sold or placed in liquidation). Nevertheless, none of these is guaranteed.

1.2 Measures of Price-Changes

The value of a financial asset is the current value of the total expected future cash flow from that asset. Finding the best function of the price for performing statistical tests is frequently a problem. There are different measures of price-changes, each one having its advantages and disadvantages and the most common are the following:[2]

1. *Linear price-change:*

$$\Delta x[t, t - \Delta t] = x[t] - x[t - \Delta t] \tag{1.1}$$

   The advantage of this approach is that it is linear. The disadvantage is that changes in the time scale have a big impact on this formula.

2. *Discounted or de-trended price-change:*

$$\Delta x^{(F)}[t, t - \Delta t] = F[t]x[t] - F[t - \Delta t]x[t - \Delta t] \tag{1.2}$$

[2] $x[t]$ denotes the price of a financial asset at time t

The advantage of this approach is that it does not involve nonlinear transformations and that the factor $F[t]$ attempts to remove the effects of inflation or deterministic biases. The disadvantage is that there is no unique choice for $F[t]$.

3. *Return:*

$$R[t, t - \Delta t] = \frac{\Delta x[t, t - \Delta t]}{x[t - \Delta t]} = \frac{x[t] - x[t, t - \Delta t]}{x[t - \Delta t]} \tag{1.3}$$

The advantage of this approach is that it provides a direct percentage of gain or loss for a given period of time. The disadvantage is that the returns are sensitive to scale changes in the long run when they increase or decrease substantially.

4. *Log-return:*

$$z[t, t - \Delta t] = \log \frac{x[t]}{x[t - \Delta t]} = \log x[t] - \log x[t - \Delta t] \tag{1.4}$$

The advantage of this approach is that it accounts for scale changes. One disadvantage is that it introduces a nonlinear transformation, which could affect the statistical properties of a stochastic process.

It can be easily shown that the indicators of price-changes defined above are approximately equal for high-frequency financial data. For analysis of data over longer time periods, the most commonly used measures of price-changes are returns and log-returns.[3]

[3]The remaining of this thesis uses the term "return" in order to refer to the logarithmic returns.

1.3 The Efficient Market Hypothesis and the Random Walk Model

Efficient Market Hypothesis (EMH) is the most important assumption in the financial market concerning stock prices. Formulated by Eugene Fama in 1970, the EMH assumes that at any given time a stock price fully reflects all available information about that particular stock.

This will occur when transactions costs are non-existent, all investors have access to the current information, and all investors agree on the implications of the available information. Depending on the content of the information available to investors, there are three forms of EMH:[4]

1. *The weak form*: it asserts that current prices reflect all information available from historical data.

2. *The semi-strong form*: it asserts that current prices reflect all publicly available information.

3. *The strong form*: it asserts that current prices reflect all currently known information (both private and public).

The weak form of the EMH implies that current changes in price are independent of past changes in price. This means that price-changes in the stock market can be compared to outcomes of a coin toss, since it is impossible to forecast its result based on past outcomes. In order to describe this behavior of financial time series, Bachelier proposed the random walk model in 1900 (see Davis and Etheridge, 2006) .[5] This model makes three important claims, which are summarized below:

[4]See Malkiel, 1989.

[5]For a mathematical model, see Appendix A.

1. The conditional expectation of the next observation, given all the past observations, is equal to the last observation;
2. The next observation is independent of past observations;
3. All changes, taken together from small to large, follow a normal distribution.

Though these three claims are the main assumptions of standard finance theory, evidence shows that they fail to hold in practice. Many studies have shown that price-changes in the stock market follow a distribution that exhibits excess kurtosis, with heavier tails than in a normal distribution (Mauboussin, 2002; Mantegna and Stanley, 1995; Pagan, 1996; Lux, 2006).[6] This finding contradicts the Central Limit Theorem, which states that the sum of random independent and identically distributed variables with finite mean and variance will asymptotically converge to a normal distribution.[7] Therefore, the assumption that price changes in the stock market are independent and identically distributed is also questionable.

Although independence of price-changes is consistent with theoretical arguments that exclude "free lunches" in the stock market (Fama, 1970), data analysis shows that the price-changes in the stock market may not be independent. While independent variables are always uncorrelated, it is not true that uncorrelated variables are always independent. Mandelbrot and Hudson (2004) offer a good explanation of this paradox:

> The key to this paradox lies in the distinction between the size and the direction of price changes. Suppose that the direction is uncorrelated with the past: The fact that prices fell yesterday does not make them more likely to fall today. It remains possible for the absolute changes to be dependent: A 10 percent fall yesterday may well increase the odds of another 10 percent mode today—but provide no advance way of telling

[6]The definition of kurtosis can be found in Appendix A.
[7]The mathematical formulation of the Central Limit Theorem can also be found in Appendix A.

> whether it will be up or down. If so, the correlation vanishes, in spite of strong dependence. Large price changes tend to be followed by more large changes, positive or negative. Small changes tend to be followed by more small changes. Volatility clusters.

Moreover, studies have showed that the random walk hypothesis is not supported by data. Craig MacKinlay, John Y. Campbell, and Andrew W. Lo (1997) reached the conclusion that "financial asset returns are predictable to some degree" since they incorporate a long-term memory component. They also conclude that "free lunch plans are ruled out" although there is a chance of getting a "free lunch" from time to time.

Another important assumption made by the EMH makes is the rationality of traders. According to Fama (1965), a market that is considered to be efficient is defined as "a market where there are large numbers of rational, profit-maximizers [i.e. the traders] actively competing with each trying to predict future market values of individual securities." This view is now being challenged by a emerging area of research known as behavioral finance. This new field supposed the non-rationality of traders, and studies the way in which individuals make decisions that deviate from perfect rational behavior predicted by the EMH.

All this suggests that the EMH needs to be replaced with a new theoretical framework that can explain the real market in a better way. According to recent research, one of the main reasons why the standard finance theory does not hold in the real market is because it assumes that, in the equilibrium state, the effects of a shock are proportional to its size. Therefore, small shocks would never have dramatic consequences, and large fluctuations are very unlikely to occur—since they would be triggered by catastrophic events which are not very frequent.

Chapter 2

Self-organized Criticality, The Sandpile Model and Power Laws

2.1 What Is Self-Organized Criticality?

A new theory that tries to account for the high volatility of the stock market argues that the stock market can be regarded as a complex dynamic system, as described by Wang and Hui (2001):

> Financial markets are typical complex systems in which the large-scale dynamical properties are dependent on the evolution of a large number of nonlinear-coupled systems.

A direct function of the interactions among the agents in a complex system is called self-organized criticality. Although a rigorous definition of Self-Organized Criticality (SOC) does not exist, we are familiar with some of its characteristics. One important chactacteristic is that the relationship between cause and effect in these systems might not be linear. Applied to the field of economics, this theory suggests that changes in interest rates and taxes are only triggering factors in stock market crashes, and not the fundamental causes. The existence of a speculative bubble could take the economy into an unstable phase in which the stock market price has reached an unsustainable pace. This unstable phase, which can be triggered by any small disturbance or process, facilitates the emergence of a crash. This way, the combined behavior of traders could gradually lead to increases in the stock market prices and eventually a crash.

2.2 The BTW Sandpile Model

Bak, Tang, and Wiesenfeld (1987) were the first to introduce the concept of SOC in order to explain the behavior of their sandpile model. The dynamics of this model are very simple. We start with a two-dimensional $n \times n$ lattice, with each site of the lattice having a certain amount of sand, denoted $h(x, y)$.[1] A grain of sand is dropped at a random vertex (x, y) such that $h(x, y) \rightarrow h(x, y) + 1$. A site fires when the number of grains accumulated at a specific site exceeds a certain threshold value: $h(x, y) \geq h_c$, where h_c denotes the threshold value at vertex (x, y), and the vertex (x, y) loses four grains of sand: $h(x, y) \rightarrow h(x, y) - 4$ which are redistributed to the adjacent sites: $h(x \pm 1, y) \rightarrow h(x \pm 1, y) + 1$ and $h(x, y \pm 1) \rightarrow h(x, y \pm 1) + 1$. Vertices on the edge of the lattice also lose four grains, spilling some off the edge, i.e. spilling some into the sink. An avalanche is the consecutive firing of many vertices. Avalanches are caused by a domino effect, in which a single grain of sand can cause one or more vertices to fire. An important characteristic of this model is that the accumulation of small grains could cause large avalanches, so searching for the cause of a big event could be very difficult.

[1] A Java applet that simulates the dynamics of the BTW sandpile model can be found at http://www.cmth.bnl.gov/~maslov/Sandpile.htm

2.3 Power Law Distributions

2.4 Mathematical Framework of Power Laws

Definition 2.4.1. A non-negative random variable X is said to have a power law behavior if there exist constants $\alpha > 0$ and $c > 0$ such that the probability density $p(x)$ follows:[2]

$p(x) \sim \frac{c}{x^\alpha}$

for $x \to \infty$ with α being the exponent.

Taking the logarithm on both sides shows that the formula above asymptotically approaches a straight line:

$\log(p(x)) \sim \log c - \alpha \log x.$

2.5 A Brief History of Power Laws

A key characteristic of SOC discovered by Bak, Tang, and Wiesenfeld (1987) is a power law distribution (also called heavy-tail distribution, Pareto distribution, Zipfian distribution, etc.) for the size and lifetime of avalanches in their sandpile model.[3] Power law distributions were also found in a lot of different areas, like the population in cities all over the world, frequencies of words in literature, the strength of earthquakes, the wealth of individuals, the size of companies, the spread of forest fires, and the extinction of biological species.

The first power law distribution discovered in the field of economics dates back to the nineteenth century when the Italian economist Vilfredo Pareto studied the allocation of wealth among individuals. His study showed that around 80% of the society's wealth was controlled by 20% of the population, contrary to what we would

[2]Adapted from Gabaix et al. (2006).
[3]For a history of power law distributions, see Mitzenmacher, 2004

have expected from a normal distribution. Many years later, Benoit Mandelbrot (1963) studied the variation of cotton prices from month to month over several years and concluded that the tails of this distribution followed a Pareto distribution with the exponent close to 1.7. The research done afterwards on wheat prices, interest rates and railroad stocks led to the speculation of a universal scale-free law involving the prices of different commodities.[4]

The possibility of this universal power-law concerning the tails of the distribution of short-term stock market returns interested many economists and physicists, being crucial for developing risk management methods.[5] Recent research (Gopikrishnan et al., 1998; Plerou et al., 1999) shows that the tails of the cumulative return distribution of the price of individual stocks and market indices follow a power law distribution, with the exponent approximately equal to 3—a property called "the inverse cubic law".[6] This behavior was found not only in developed countries, but also in some developing countries like China (Gu, Chen and Zhou, 2008), India (Pan and Sinha, 2008), Korea (Kim et al., 2005) and Mexico (Coronel-Brizio et al., 2005). This latter discovery is quite surprising, taking into account that emerging markets are subject to severe macroeconomic instabilities, and hence, are not expected to follow the same market dynamics as the more developed countries do. Other studies show that markets might indeed exhibit a power law behavior in the tails, however the exponent was not necessarily equal to 3, but between 2 and 4 (Pagan, 1996; Longin, 1996; Guillaume et al., 1997). A few models were developed in order to explain the emergence of power law distributions in the stock market returns, one of which is the dynamic multi-agent model based on herding behavior.

[4]The scale-free behavior refers to the fact that, as shown in the previous section, a power law distribution has the property that the probability density function in a log-log plot is a straight line.

[5]Short-term refers to a time interval between one minute and a few days.

[6]Actually, the fact that the tails of the distribution of individual stocks obey a power law decay would immediately result in the market indices also obeying a power law since these market indices reflect the aggregate value of many individual stocks and also because most of the prices of stocks that are part of the same index tend to move together.

The ubiquity of this power law is indeed questionable, and there are other hypothesis concerning the tails of the distribution of stock market returns, including the exponential distribution (Matia et al., 2004; Dragulescu and Yakovenko, 2002; Oh, Um and Kim, 2006; Kaizoji and Kaizoji, 2003; Silva et al., 2004) and the stretched exponential distribution (Laherrere and Sornette, 1998; Gu et al., 2008). The problem with these discoveries is that the exponentially decaying tails are indicative of the presence of a scale—hence the universal scale-free behavior of stock markets would fail miserably. Moreover, the exponential function exhibits thinner tails, and thus, there would be less extreme price fluctuations than the ones predicted by a power law. A recent study by Malevergne, Pisarenko, and Sornette (2005) concluded that the tails decay slower than any stretched exponential distribution, but faster than any power law distribution.

Kaizoji (2005) investigated the returns in the Japanese stock market and concluded that they display a different shape in the periods of stagnation than in the periods of boom. He discovered that during booms, the tails are characterized by a power law with exponent 3, while during stagnations, the tails are characterized by an exponential distribution with the scale parameter close to 1. Furthermore, he explained this behavior by modeling the stock market as a system consisting of interactions among traders.

Although initial research focused only on returns, power laws have also been found to characterize the number of shares traded and the number of trades in the stock market.[7] Gopikrishnan et al. (2000) find that high-frequency data from the US stock market exhibit the following characteristics:

1. The trading volume (V), which denotes the number of shares traded, is characterized by the following behavior: $P[V > x] \sim x^{-1.5}$

2. The number of trades (N) also follows a power law: $P[N > x] \sim x^{-3.4}$

[7] Note that these two quantities are closely related.

The fact that so many components of the stock market follow a power law could indicate a common underlying mechanism that defines the dynamics in the stock market. One important factor that could connect stock market returns with the number of trades and the volume of trades is information. The following chapters consider the stock market as a communication network where traders interact with each other, and where information is valuable and strongly correlated with returns. Some models of interaction among traders are considered in Chapter 3 and a new model of information is developed in Chapter 4.

Chapter 3

Herding Behavior

3.1 What Does Herding Behavior Refer to?

Although different models have been created in order to explain the existence of fat tails in stock market returns, most of them are not based on any market phenomenon that would generate this data. In order to relate the high volatility of the stock market to the psychology of traders, a new area of research was needed—behavioral finance. This new field, briefly mentioned in Chapter 1, gives a new perspective on the way investors make decisions. Shiller (2000) argues that large fluctuations of prices could be the cause of herding behavior among traders. He believes that herding behavior in the stock market during booms and crashes does not necessarily imply irrationality on the side of the traders, since even rational people could take into account the beliefs of other people. One explanation he gives for herding behavior is that people who are in touch with each other often think in the same way. This could be possible when people are reacting to the same information at the same time. Another cause for herding behavior is the possibility of conflicting ideas coexisting inside a person's mind, in which case people might be acting contrary to their beliefs. Shiller points out that the existence of contradictory beliefs could be the effect of experts having previously endorsed both points of view.

Following Shiller's observations, the literature on herding behavior has developed a lot over recent years in a multitude of domains. Parker and Prechter (2005) divide the theoretical approaches in different categories, according to the different fields it has been applied to: social psychology, informational theory, econophysics, medicine and socio-economics. Two well-known models of herding behavior dealing with the informational theory of herding were proposed by Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992). Both of these models assume that each individual makes a decision by taking into account their private knowledge and the decisions of the individuals preceding them. The sequential character of herding from these models is a serious disadvantage: the natural way of ordering agents, through which each one only imitates the actions of preceding traders, does not correspond to reality.

Another preoccupation of researchers has been trying to connect herding behavior to the existence of power laws. One of the first models to attempt this was the Bak, Paczuski and Shubik model (1996). Although the p.d.f. of the returns exhibits excessive kurtosis, the model does not necessarily imply the existence of power laws. Cont and Bouchaud (2000) solve the problem of the sequential character of herding behavior, and develop a model in which the formation of clusters is taking place due to the formation of random connections between traders. They also show that the distribution of clusters follows a power law.[1]

Recent research has been dedicated to econophysics models, which are based on catastrophe theory and self-organized criticality, and are very similar to informational theory models—the only difference being that they model the dynamics of bubbles endogenously. Before talking about these econophysics models, some knowledge about informational theory models is required.

[1]More details on the last two models are given in the following section.

3.2 Three Important Models

3.2.1 Bak-Paczuski-Shubik Model

Bak, Paczuski and Shubik (1996) developed a model based on the interactions among two types of traders (fundamentalist and noise) in order to account for the large variations in stock market prices. The fundamentalist traders are agents who optimize their own utility functions by making a fundamental analysis of the stocks. Noise traders study market dynamics and rely on noise rather than information. A common belief about noise investors is that they follow trends, overreact to news and have poor timing.

The interactions between the agents in this model are defined by the following framework:

- There is only one type of stock. Each agent can own 0 or 1 shares. The share provides the owner a dividend at a specific interval of time. This dividend pays A with probability ρ and pays B with probability $(1 - \rho)$.

- All agents have enough money to buy a share and the money they are left with after the transaction is held in an account bearing an interest rate i.

- There are $N - K$ noise traders, K traditional traders and $N/2$ shares available for the traders to buy. Thus, half of the traders are potential buyers and the other half potential sellers.

- Each potential buyer (an agent with 0 shares) makes a bid at a certain price denoted p_b and each potential seller (agent with 1 share) offers his share at a certain price denoted p_s. At each step, a trader is chosen randomly to be the one who performs a market operation. The agent will update his price according to his strategy.

Following these rules, a transaction takes place when an overlap occurs between buyer and seller: the trader sells his share to the highest bidder or buys the share from the lowest seller. This way, the buyer becomes a potential seller and the seller becomes a potential buyer, i.e. each of them change their prices. The strategy used for changing their prices depends on the type of trader the agent is. If the agent is a traditionalist (also called rational) trader, then he will assess the fundamental value of the company and then choose to buy or sell depending on the interest rate he is offered, and his utility function which takes the form:

$$U = \nu \min[A, B] + (1 - \nu)(\rho A + (1 - \rho)B), \tag{3.1}$$

where ν represents each agent's risk preference and it is a value between 1 (in which case we are dealing with extreme risk aversion) and 0 (the case of risk neutrality). Therefore, the rational agent will make a decision to buy a stock if $\frac{U}{p_s} > i + \Delta_1$ and will decide to sell a stock if $\frac{U}{p_b} < i + \Delta_2$. Here, the parameters Δ_1 and Δ_2 represent the decision stickiness. On the other hand, if the agent is a noise trader, he will ignore the dividends and the fundamentals and instead examine market trends and the behavior of other traders. There are three strategies that the noise trader could use in order to set a new price:

1. Choose it at random;
2. Make a guess of the market price and bid towards it;
3. Pick another agent at random and copy his price.

Furthermore, Bak, Paczuski and Shubik perform some computer stimulations of the model and they arrive at several conclusions. First of all, they take two extreme cases, where there is only one type of trader—either rational or noise. In the first case when there are only rational traders in the market ($N = K \neq 0$), the market stabilizes after some time and no further trade occurs. In the second case with only noise traders ($K = 0$ and $N \neq 0$), the price changes are dramatic and several small effects of fat tails of return distribution appear.

A third possibility is the one with both types of agents trading in the market, with noise traders imitating other traders chosen at random. Because of the randomness of their imitation, the noise traders could imitate either other noise traders, or rational traders, since they are unaware of which type of trader one has chosen to imitate. Numerical simulations of these models show that when 2% of the traders are rational traders, they would be driven out of the market and the noise traders left in the market would imitate each other. In the end, this herding behavior will bring about unreasonably high prices in the market ("bubble"). Another stimulation shows that with 20% of rational traders in the market only small deviations of the prices occur.

Although this model simulates real market conditions in a faithful manner by exhibiting a fat-tailed behavior of returns, it is complex and it contains too many components that are hard to trace.

3.2.2 Cont-Bouchaud Model

Cont and Bouchaud (2000) investigated the influence of herding behavior on the existence of heavy tails in the distribution of returns. They considered a model of determining price largely used in econophysics. The model consists of N agents trading in a single asset. The demand for stock of an agent i is represented by ϕ_i and it could take the values $+1$, -1 and 0, depending on the agent's intentions. Thus, the demand for stock will take the value $+1$ if the agent wants to buy a stock, -1 if he is wants to sell a stock and 0 if he does not want to trade. In equilibrium, the sum of all the ϕ_i variables will have to add to zero. Outside of equilibrium, the price adjustment process is given by the following formula:

$$\Delta x = x(t+1) - x(t) = \frac{1}{\lambda} \sum_{i=1}^{N} \phi_i(t), \tag{3.2}$$

where λ is a factor measuring the liquidity in the market and is defined to be "the excess demand needed to move the price by one unit." If ϕ_i is an i.i.d. random variable, we could apply the Central Limit Theorem for N large enough and Δx would have an approximated Gaussian distribution. Empirical evidence discussed in Chapter 1 tells us that this is not true. Moreover, if the demands of investors are not i.i.d. variables then heavy-tails could exist in the return distribution.

The Cont-Bouchaud model further assumes that agents are grouped into clusters and that investors from the same cluster take the same market decision. The demand equation is rewritten as:

$$\Delta x = \frac{1}{\lambda} \sum_{\alpha=1}^{k} W_\lambda \phi_\lambda(t), \tag{3.3}$$

where k is the number of clusters, W_α is the size of a cluster α and ϕ_α is the demand of the agents belonging to the same cluster α (the same for all agents in the cluster).

Since the distribution of the change in prices depends on the size of each cluster, the formation of coalitions among agents plays an important role. For a pair of agents i and j, Cont and Bouchaud denote p to be the probability that the agents are part of the same cluster.[2] On average, an agent will be linked to $(N-1)p$ other agents. Furthermore, taking $c = N * p$ to be the average number of friends makes $(N-1)/p$ have a finite limit. Therefore the distribution of cluster sizes, determined by the parameter c, eventually determines the distribution of returns.

The authors conclude that for c close but smaller than 1, the cluster size has a p.d.f. that in the limit of large W is cut off by an exponential tail and looks like:

[2]Since all links are equally probable, the choice of p is independent of the choice of i and j.

$$p(W) \sim W^{-5/2} \exp\left(\frac{(1-c)W}{W_0}\right) \tag{3.4}$$

In the case that $c = 1$, the tail of the distribution is a power-law in the limit of large W and is given by the following formula:

$$p(W) \sim W^{-5/2} \tag{3.5}$$

One important finding is that the return distribution can have an arbitrary large kurtosis provided c is close to 1. In this case, one agent will be linked with another agent, and a finite fraction of the market will be part of the same cluster, which could lead to a crash. Also, the state where c converges to 1 is leading to scaling-laws and heavy tails since the clusters become increasingly larger.

Although this model accounts for many phenomena encountered in the real market, there are some aspects that it does not account for. The critiques of this model argue that it is hard to know exactly how traders exchange information and also that c is not easy to estimate. Another problem is the fact that the links between traders might change over time.

Percolation Theory

Percolation theory refers to the study of the connectivity of networks. In the case of a two dimensional square lattice, the percolation theory studies the degree of connectivity among its sites. The percolation threshold is defined to be the average connectivity between the sites of the lattice. Its value is between 0 and 1, with 0 when all the sites are isolated from each other and 1 when all the sites are connected to the maximum number of neighbors possible.

If we take p to represent the average connectivity among sites, then the percolation threshold is the minimum value of the parameter—denoted p_c—at which there exists a connected path that connects the top and the bottom of the lattice. For the simple two-dimensional lattice chosen, the percolation threshold is $p_c = 0.593$.[3]

The Cont-Bouchaud model is structured on the basis of the percolation theory on a random graph of size N in which the aggregations of traders are modeled as clusters (defined as neighboring sites that are occupied) and the links among traders are the connections between different vertices of the graph.

3.2.3 Stauffer-Sornette Model

Following the Cont-Bouchaud model, Stauffer and Sornette (1999) introduce a percolation model to describe herding behavior in the stock market.[4] Their model attempts to improve the Cont-Bouchaud model by defining the parameter p—denoting the connectivity among traders—as a random value. This parameter is essentially measuring the degree of herding behavior existent in the system: as the value of p gets bigger, herding behavior becomes stronger. Cont and Bouchaud (2000) consider this parameter fixed at the percolation threshold, but in reality the number of connections among traders could change over time.

The authors consider a square matrix on which the traders represent the sites of the matrix and are distributed with degree of connectivity p, varying at random at each time step from 1 to 59. The latter percentage corresponds to the critical value $p_c = 0.593$ for a square lattice. When p reaches the value 0.593—its critical value—a stock market crash emerges and afterwards the parameter p is reinitialized to a lower value. For each value of p, one percent of the traders move to a random neighboring site, and this procedure is iterated 1000 times. Similar to the Cont-Bouchaud model, this model offers three possibilities for each cluster in each time interval: buy, sell or decide not to trade. The probability that a cluster sells a stock is considered to be a,

[3] This value is very important in the Stauffer-Sornette model discussed in the following section.

[4] Although the Cont and Bouchaud paper was published in 2000, its first draft was ready in 1997, which made possible for Stauffer and Sornette to read it.

which is also the probability that a cluster chooses to buy a stock. This parameter is taken to be ranging between 0 and 0.5. For each cluster taken separately, the decisions to buy or sell are aggregated and the total number of simulations is averaged to give smooth results. The results show that the distribution of returns exhibits a power law with exponent 2.5, slightly different from the exponent 4 observed in previous research.

In order to get a higher degree of accuracy, Sornette and Stauffer take a different approach. The fact that the parameter a was considered to be a free parameter could be a problem since research shows that important investors (such as mutual funds and retirements funds) tend to trade less than smaller investors. Moreover, the decision to buy or sell are also contingent upon the number of traders that form a cluster, since the decision is reached only when the majority agrees. Therefore, the parameter a is taken to be dependent on the cluster size as follows: $a = \frac{0.5}{\sqrt{s}}$, where s represents the cluster size which is proportional to the amount traded. With these new activity parameter, the distribution of returns is found to follow a power law with an exponent around 3.5, much closer to the expected exponent of 4.

Consequently, the three models of herding behavior considered in this chapter seem to be indicating a power law behavior related to the phenomenon of herding. Since herding behavior is strongly related to sharing information, an information transmission model is developed in the Chapter 4. This model is expected to follow a power law behavior under certain circumstances to be determined in the Chapter 5.

Chapter 4

An Information Transmission Model

4.1 Power-law Behavior

As seen in the previous chapters, a system possessing a power-law behavior consists of a very large number of parts with a very limited influence and a very small number of parts that dominate. Moreover, such a system exhibits scale invariance since the smaller components of the system have the same characteristics as larger ones. Given the general formula of a power-law, $f(x) = cx^{\alpha}$, scale invariance can be derived by scaling the variable x by a constant factor:

$$f(kx) = k(cx)^{\alpha} = c^{\alpha} f(x). \tag{4.1}$$

This way, we can see that $f(kx)$ can be obtained by multiplying $f(x)$ by a constant, which is the reason why we expect a straight line when the logarithm is taken on both sides of the previous equation.

4.2 The Connection between Power Laws and Information

Since herding behavior is strongly connected to information transmission, one question that needs to be asked is whether or not there is any connection between information and power-law behavior. Large events can be set off by the flow of information between people, without any initial large shock. According to Srinivasa (2006), the configuration of the information network plays an important role and the more densely connected the network is, the faster the propagation of information. Moreover, each individual inside the network has his own threshold for adopting a new idea. This threshold together with the degree of connectivity among people could cause the informational cascades, similar to the avalanches in the sandpile model.

In order to understand how the network of traders is established, some graph theory is required.[1]

4.3 Brief Introduction to Graph Theory

Definition 4.3.1. A graph is a pair $G = (V, E)$ with the elements of V being the vertices and the elements of E being the edges, each edge defined as a set of two vertices.

Definition 4.3.2. A path is a non-empty graph $P = (V, E)$ of the form

$$V = \{x_0, x_1, \ldots, x_k\} \qquad E = \{x_0x_1, x_1x_2, \ldots, x_{k-1}x_k\},$$

where the x_i are all distinct.

Definition 4.3.3. Two vertices x, y of G are neighbors if there is an edge connecting them.

[1] Adapted from Diestel (2005).

Definition 4.3.4. The degree of a vertex v is the number of edges at v, and it is equal to the number of neighbors of v.

Definition 4.3.5. A graph with all the vertices of the same degree is called regular. A regular graph of degree d is referred to as a d-regular graph.

Definition 4.3.6. A directed graph is a pair (V, E) of vertices and edges together with two maps $init : E \to V$ and $ter : E \to V$ assigning to every edge e an initial vertex $init(e)$ and a terminal vertex $ter(e)$. The edge e is said to be directed from $init(e)$ to $ter(e)$.

Remark 4.3.7. A directed graph might have several edges between the same two vertices x and y and are called multiple edges. If these multiple edges have the same direction, they are parallel. If $init(e) = ter(e)$, the edge is called a loop.

Definition 4.3.8. Each vertex of a directed graph has an in-degree and an out-degree. The in-degree of a vertex v corresponds to the number of edges with v as a terminal vertex. The out-degree of a vertex is the number of edges for which v is a initial vertex.

Definition 4.3.9. A graph or directed graph in which multiple edges are allowed is sometimes called a multigraph.

Definition 4.3.10. A weighted edge of a graph is an edge e with a corresponding weight, which is a natural number denoted $w(e)$ interpreted as the number of multiple edges between the two vertices that correspond to that edge. A weighted graph is a graph whose edges are weighted.

4.4 Dhar's Abelian Sandpile Model

Dhar (1990) developed a model based on the BTW model discussed in Chapter 2. The model is defined on a directed multigraph G with edges weighted by non-negative integers, where each vertex has an out-degree and an in-degree as defined in the previous section. The sink is the special vertex for which the out-degree is zero, assuming that there is a path from each vertex to the sink.[2] A configuration c on the graph G is defined to be the assignation of a natural number to each vertex, corresponding to a certain number of grains of sand. The amount of sand on each vertex c is denoted $c(v)$. A vertex is unstable if the number of grains of sand at that vertex is greater or equal to the out-degree of the vertex:

$$c(v) \geq out\_deg(v) \tag{4.2}$$

Each unstable vertex fires (or topples) and it sends one grain of sand to each of its neighboring vertices to which there are outgoing edges so that in the end $out\_deg(v)$ grains of sand are sent.[3] A sequence of firings is called an avalanche. Any configuration can become stable after a series of avalanches.

4.5 The Model

The model developed is very similar to the one developed by Lee (1998) and is described in the following subsections.

4.5.1 The Market

The model considered attempts to simulate the communication network of traders in the stock market in the period following an informational cascade. This informational cascade refers to a situation in which a trader ignores his private information and decides to imitate the other traders. As suggested by Lee (1998), an important cause of this behavior is the high transaction costs that prevent traders from sharing their

[2] Thus, it does not have any outgoing edges.
[3] Note that the sink will never fire.

private information when they trade in a sequential fashion. In order to facilitate the simulation of this model, it is assumed that transaction costs stay the same throughout the trading periods.

The fact that stock prices in this market have increased a lot in the previous periods causes the traders to question their previous investment decisions. Hidden information starts building up as traders start regretting their decisions and one small shock could be enough to create the reverse of an informational avalanche—called an informational cascade, a situation in which people can no longer disregard their private information and start acting according to it (i.e. sell or buy stocks). Since the market considered in this model is one in which prices are artificially high due to aggregation of information failure, we assume that all the private information in this market leads to selling stocks.

An important observation to be made is that the dynamics of an informational cascade can be easily simulated by the sandpile model in which the network of traders is represented by a graph where the vertices are individual traders and the edges are the links between them. The main difference from the model developed by Lee (1998) is that this model also has a sink, characteristic to be explained in the next subsection.

4.5.2 Transmission of Information

This model's information dynamics are described by the grains of sand that move from one vertex to another or that are dropped at random vertices. The grains of sand dropped at random represent either the news that a trader receives from outside the network of traders or just increased willingness to share private information that could start an informational cascade. Another way in which a trader could receive a piece of information is in the case his neighbor has decided to sell a stock based on his private information. This decision is incorporated by the firing of that specific vertex which is followed by grains of sand being sent to the neighbors. Therefore, receiving a grain of sand from a neighbor corresponds to receiving a piece of information about selling. We also consider that all traders hold either the same stock or stocks whose prices are correlated with each other, so that each piece of information is relevant. In order to make a decision, each trader takes into account his private information and also the signals received from his neighbors. For simplicity, we consider all pieces of information to be the same. Moreover, the fact that a trader might have previosly received a piece of information does not make him immune to receiving the same piece of information again.

The initial sand configuration represents the private information that each trader initially has. Once a trader has shared a piece of information, he is waiting to receive new pieces of information, as the old ones lose their importance. After a trader has shared his piece of information with his neighbors (i.e. after that vertex fires), there are no grains of sand in that vertex, suggesting that some private information about selling no longer exists until more grains are accumulated. The firing of a vertex here is considered to be equivalent to sharing the private information which is equivalent to selling of stocks. This model is attempting to simulate only the flow of information in the market and is not concerned with the stock prices, although they are the underlying reasons that determine traders to transmit information.

4.5.3 Decision Rules

The decision that the traders in the market can take is to either sell a stock (which also implies sending a signal to its neighbors) or to wait for a limited period of time before selling. Sharing a piece of information about selling a stock corresponds to the firing of a vertex while waiting is just the period of time it takes a vertex to fire while it is accumulating grains of sand. Each trader corresponding to a vertex v can influence a neighboring trader corresponding to a vertex v' if the following two conditions hold: $v = init(e)$ and $v' = ter(e)$.

The fact that some traders can be more influential than others is taken into account by the degree of connectivity between vertices. Thus, since information flows along an edge e from $init(e)$ to $ter(e)$, the out-degree of vertex $init(e)$ determines how influential that trader is for the trader denoted by vertex $ter(e)$. In the case that a vertex is connected to the sink, the degree of connectivity between that vertex and the sink represents two important aspects of the market:

1. There is some degree of stickiness associated with the decision rule that a trader makes based on transaction costs. A piece of information (corresponding to a single grain of sand) that an agent receives will just increase his willingness to sell as more grains of sand gather at the vertex but this will not necessarily determine an immediate firing of the vertex. A vertex will only fire immediately after receiving a piece of information if it is in its maximal stable state so that the amount of sand found at that vertex is equal to $out\_deg - 1$. Furthermore, the higher the degree of connectivity of that vertex to the sink, the higher its out-degree and the more grains of sand needed for that vertex to topple.

2. Since the grains of sand sent to the sink are lost forever, the degree of connectivity to the sink also account for the loss of information while it gets transmitted from trader to trader. This could happen because the information loses its importance (since it becomes incorporated in the new stock price) and/or because old information gets obsolete as new information becomes available. In the real market only the unimportant information is lost, but this model treats all the pieces of information in the same way.

Chapter 5

The Experiment

A modified version of professor David Perkinson's SAGE program for doing calculations in Dhar's abelian sandpile model was used for the simulations.[1] We start with the maximal stable configuration which is that configuration with $out\_deg(v) - 1$ grains of sand at each vertex v. The benefit of starting out with this configuration is that adding a grain of sand at a random vertex will make that vertex unstable right away. This implies that the system is in an unstable state and that a single grain of sand dropped from outside the system causes an avalanche. Each unstable vertex has to fire at some point in time. An important remark is that each unstable configuration reaches a stable one in a finite number of time steps as long as each vertex has a path to the sink. [2] Therefore, we could always count the number of vertices that fired, corresponding to the number of traders who decided to share their private information.

As Sornette (2003) noted, there are three ways we could measure the strength of an avalanche:

1. **Size:** the total number of firings;

2. **Area:** the total number of vertices that fired;

[1]The code can be found at http://people.reed.edu/∼davidp/412/sage-sandpiles/sandpile.sage.
[2]For a detailed proof of this fact, see Dhar (1990).

3. **Lifetime:** the duration of the avalanche, where each time step is represented by the firing of all the unstable vertices. Thus, the duration counts the number of time steps it takes the system to stabilize.

In our model, the size of an avalanche denotes the number of times the investors received a piece of information from other investors, the area denotes how many traders received the piece of information and the lifetime of an avalanche represents the severity of the information cascade and the length of time it takes for it to stabilize.

5.1 Goal

The goal of this experiment is to analyze the distribution of the strength of the avalanches obtained after running different versions of the experiments based on the model described above. Each experiment consists in plotting the area, size and lifetime of avalanches on a log-log scale using different types of networks. A power law behavior is expected to be present, at least in some of the experiments. The main question to be asked is—under what circumstances do the measurements of the strength of an avalanche follow a power law?. Therefore, the experiments in the next sections determine the type of communication network that is required in order to obtain a scale-invariant behavior.

5.2 The BTW model

We replicate the experiment that Bak, Tang and Wiesenfeld (1987) conducted using the sandpile model. Based on the results of the authors, a power law is expected to characterize the behavior of the sizes of the avalanches. In addition to the size of the avalanches, the area and lifetime are also calculated.

We start the experiment by considering an $n \times n$ undirected graph with each vertex on the boundary connected to the sink. All the non-sink vertices have degree 4, and so the corner vertices are connected to the sink by an edge of weight 2, while the other vertices on the boundary are connected to the sink by an edge of weight 1. An example of a 4×4 grid is shown in Figure 5.1.

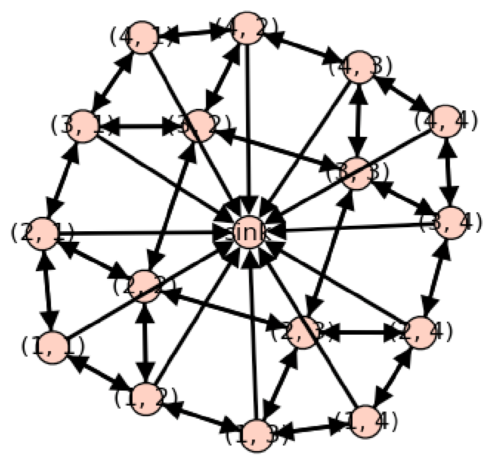


Figure 5.1: The 4×4 grid graph with a sink attached to the vertices on the borders.

The experiment goes as following:

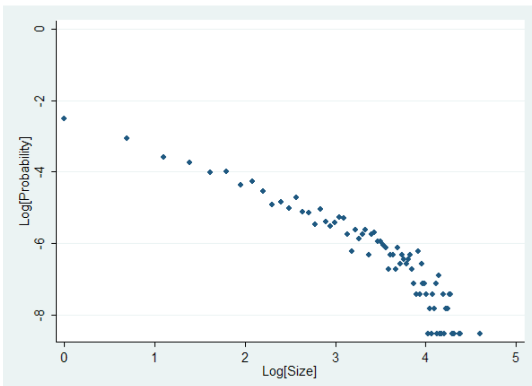
1. Set the initial sand configuration to the maximal stable configuration;
2. Drop a grain of sand at a random vertex;
3. Stabilize the system and record the size, area and lifetime of the avalanche;[3]
4. Run the experiment m times every time starting from the previously stable configuration obtained;
5. Graph the frequency distribution of the sizes, areas and lifetimes of the avalanches on a log-log scale and test whether or not they follow a power-law behavior.

[3]Note that dropping a grain of sand on a random configuration does not necessarily start an avalanche unless the vertex on which the grain of sand was dropped is unstable. In our experiment, we ignore all these cases and we regard them as the waiting time between avalanches.

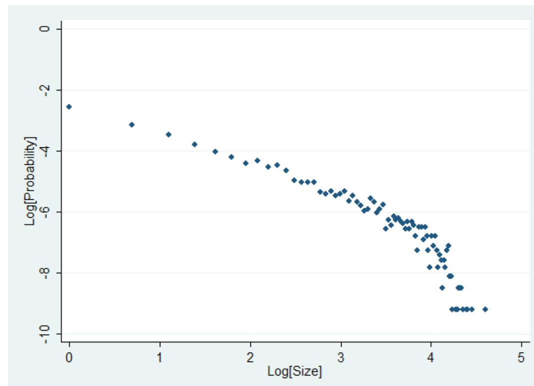
A 10×10 grid graph was chosen in order to conduct the experiments described above.[4] The experiments measuring the size of the avalanches were run 5000, 10000, 50000 and 100000 times so that we could observe the behavior of the avalanches over different runs. The log-log graphs of size of the avalanches versus the probability of obtaining a size of a certain magnitude for the different runs are shown in Figure 5.2.

The areas and lifetimes of avalanches were also computed for the 10×10 grid graph for $10,000$ runs only. Since all the experiments contain the same number of runs, we replace the probability of obtaining a value of a certain size with the number of times a value of that size was obtained. Figure 5.3 shows the log-log graphs of the distribution of avalanche areas and lifetimes for $10,000$ runs.

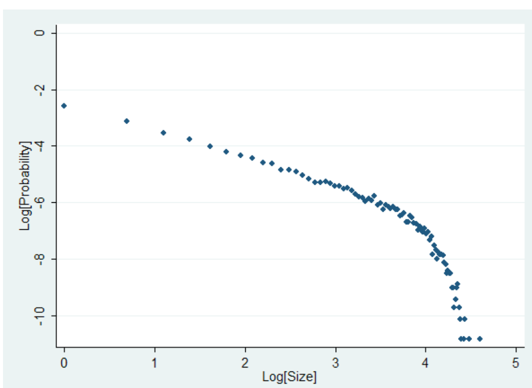
[4]This grid has 100 non-sink vertices plus one sink vertex.



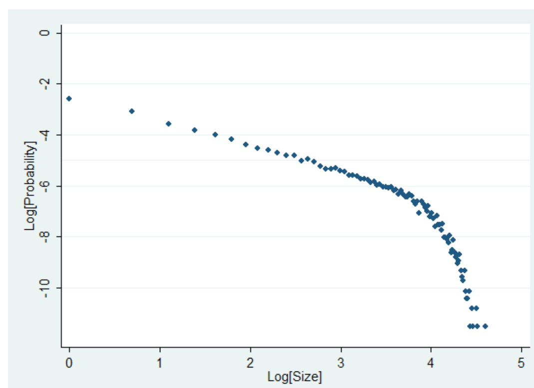
(a) 5,000 runs



(b) 10,000 runs

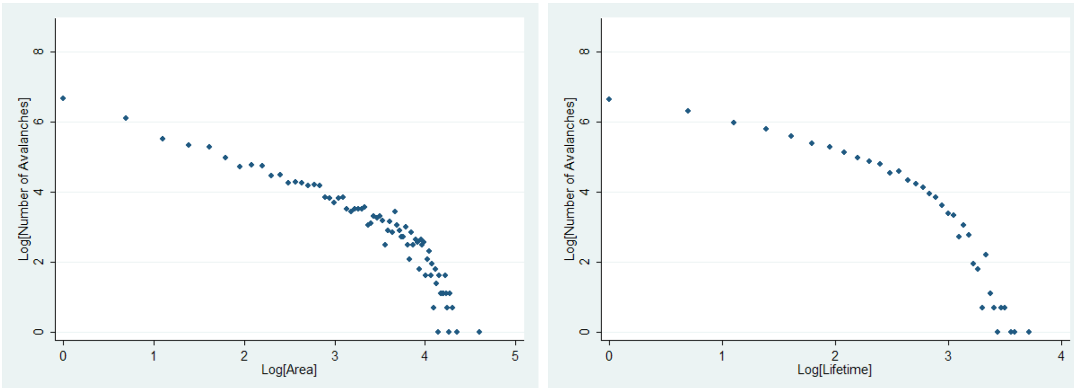


(c) 50,000 runs



(d) 100,000 runs

Figure 5.2: The distribution of avalanche sizes on a 10×10 grid graph for different runs.



(a) Area (b) Lifetime

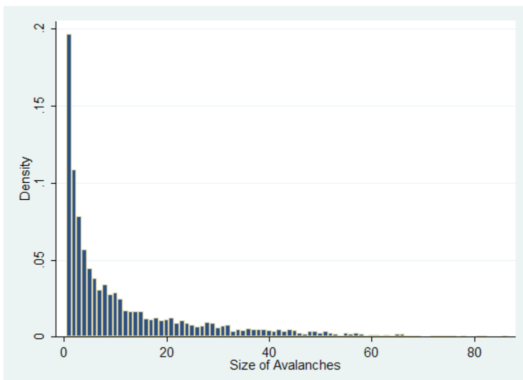
Figure 5.3: The distribution of avalanche areas and lifetimes on a 10×10 grid for $10,000$ runs.

One important observation is that fitting a straight line to any of the graphs obtained above does not work. Although the graphs seem to scale as a power law over a finite region, they are decaying exponentially for large values of the parameters (area, size or lifetime). The reason why this is happening is that the distributions in the previous figures cannot scale as the power laws in the limit $x \to \infty$ since an avalanche has a maximum size/area/lifetime each time the experiment is run. Another reason is the fact that the percentage of vertices connected to the sink decreases as the size of the grid increases, so that in a bigger grid there is a higher probability of getting bigger avalanches.[5]

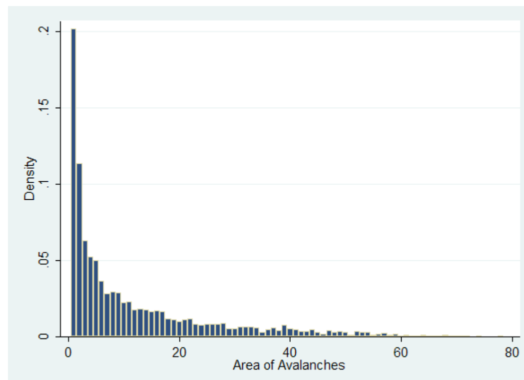
Carlson and Swindle (1995) solved the problem of the exponential decay by allowing the distribution of avalanches in the BTW model to be defined by a power law with an exponential cutoff rather than by a "pure" power law. The power law exponential cutoff follows the following behavior:

$$p(x) \sim cx^{-\alpha}e^{-\lambda}.$$

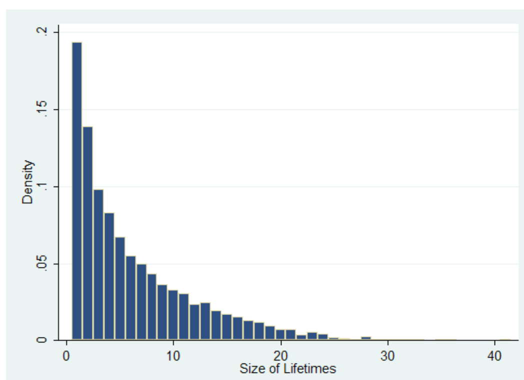
[5]This observation is just a hypothesis and needs further investigation. We know that an $n \times n$ grid would consist of $4n - 4$ vertices connected to the sink, and so when n increases, the ratio $\frac{4n-4}{n^2}$ decreases, but the magnitude of decrease is very small compared to the increase in the size of the grid.



(a) Size



(b) Area



(c) Lifetime

Figure 5.4: The histograms of avalanche sizes, areas and lifetimes on a 10×10 grid for $10,000$ runs.

In order to determine the equation that best fitted the data, we run regressions using the logarithm of number of avalanches as the independent variable and using different powers of the size of avalanches and the logarithm of size of avalanches as dependent variables. One problem encountered when examining the data was the strong multicollinearity among the dependent variables. Although multicollinearity does not have any impact on the forecast of the regressions, we solve this problem in order to understand how various independent variables impact the dependent variable. Therefore, we center the dependent variables by replacing each variable with the difference between it and its mean. The equation that was found to fit the best is the following:[6]

$$Log(\text{Number of Avalanches}) = \beta_0 + \beta_1 * Size + \beta_2 * Size^2 + \beta_3 * Log(Size) + \beta_4 * Log(Size)^2$$

This formula suggests that the cutoff obtained for the actual data is bigger than the one for a power-law with exponential cutoff distribution. The graph and the fitted curve are showed in Figure 5.5.

The histograms for the sizes, areas and lifetimes of avalanches are shown in Figure 5.4. They show a SOC behavior with fewer big avalanches and also fewer avalanches of longer duration.

In order to examine the dependence of information transmission on the type of the network of traders, different graphs were considered. The remaining experiments were conducted on graphs having 100 non-sink vertices and one sink vertex and each experiment was run $10,000$ times.

[6]The values of the coefficients obtained in the regression are not reported since we are mainly interested just in the equation in general and not on these values.

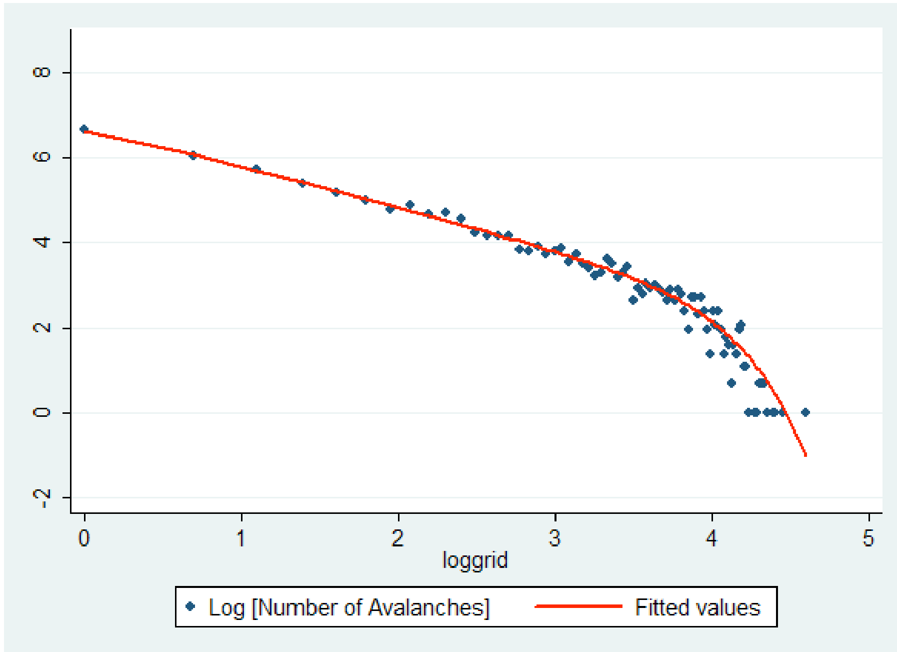


Figure 5.5: Fitted values for the sizes of avalanches.

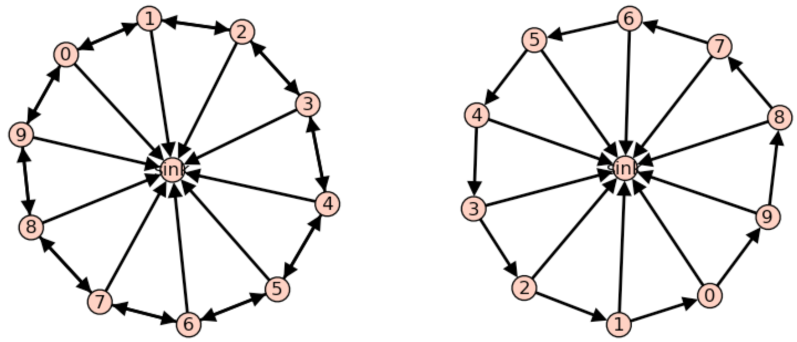
5.3 The Circular Network

A circular network is defined to be a graph with each vertex connected to exactly three other vertices: the sink and two non-sink vertices. Depending on the edges between the nodes, a circular graph could either be undirected (shown in Figure 5.6(a)) or directed (shown in Figure 5.6(b)).

5.3.1 Undirected Circular Network

An undirected circular graph implies a network where each trader—represented by a vertex v—can only be influenced by its two neighbors who are also the only sources of information the trader has. Repeating the same experiment as the one done for the grid graph, we get the graphs and histograms in Figures 5.7 and 5.8 shown below. In order to create the histograms for the circular networks, the first avalanche was ignored for obtaining a better scaling (since this was the only very big avalanche).

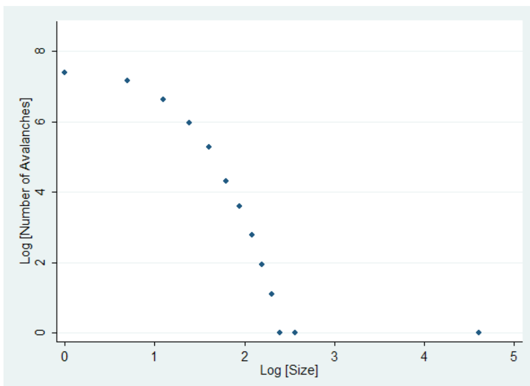
This experiment suggests that the areas and sizes for an undirected circular net-



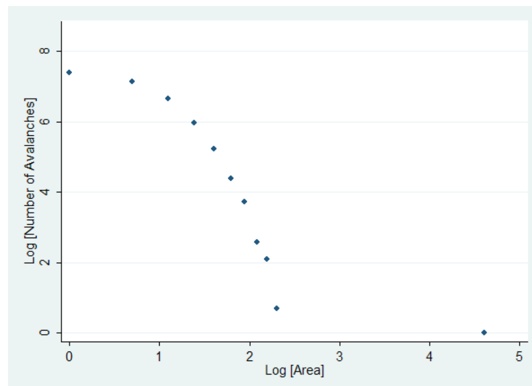
(a) Undirected (b) Directed

Figure 5.6: The graph of a circular network with 10 vertices connected to the sink.

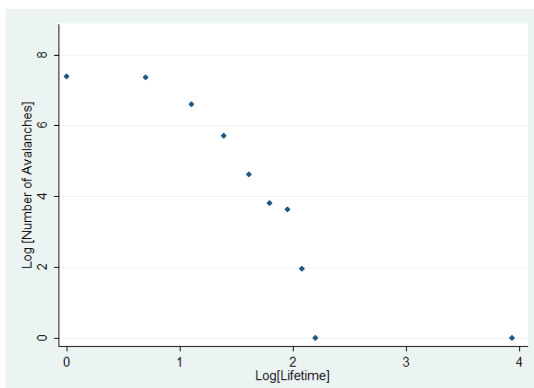
work are approximately equal and that very few vertices fire twice during the same avalanche. Moreover, the lifetimes of avalanches in this case were a lot smaller than the ones obtained for the grid graph. This suggests that the undirected circular network is very inefficient in transmission information from one vertex to another, since each person is only connected to two other people.



(a) Size

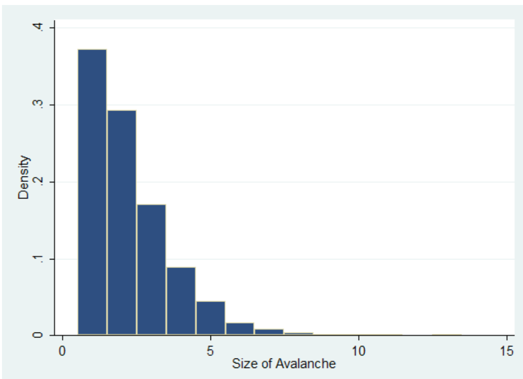


(b) Area

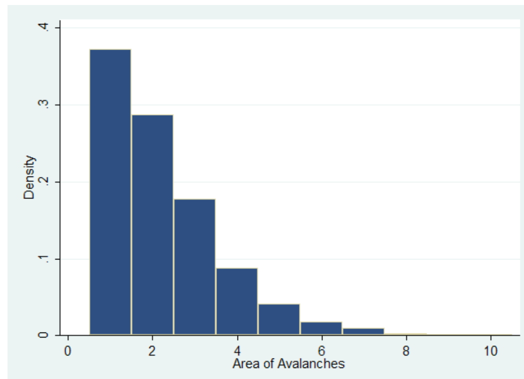


(c) Lifetime

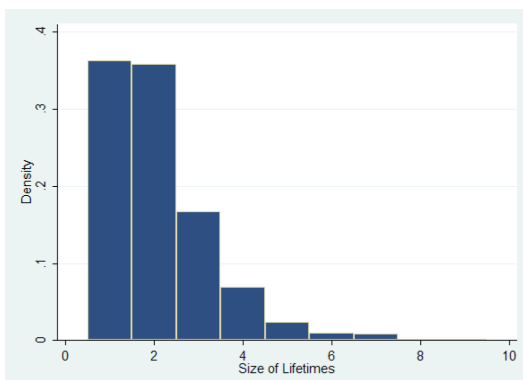
Figure 5.7: The distribution of avalanche strength (size, area, lifetime) on an undirected circular graph.



(a) Size



(b) Area



(c) Lifetime

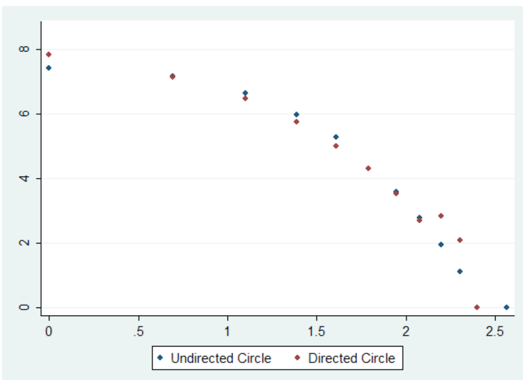
Figure 5.8: The histograms of avalanche strength (size, area, lifetime) on an undirected circular graph.

5.3.2 Directed Circular Network

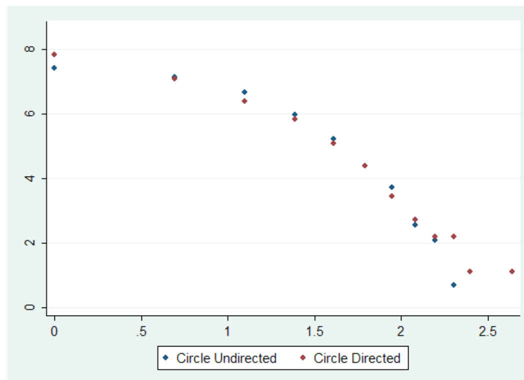
Compared to an undirected circular network, a directed network implies that each trader is only influenced by one person and can influence one person only. Moreover, the strength of the avalanche (see Figures 5.9 and 5.10) appears to depend on the degree of connectivity to the sink. This type of network simulates the case where information is transmitted in a chain manner, and where each trader gets his information from only one person. Although not a very realistic assumption, it helps establish under which circumstances a power law would emerge in a model of information transmission. We expect this network to be even less efficient in transmitting information than the undirected circular network.

We compare the sizes, areas and durations of the avalanches in the directed circular network with those in the undirected one (see Figure 5.9) and we conclude that there are no significant differences between the areas and sizes of avalanches of directed and undirected circular graphs with initial degree of connectivity to the sink of 1. This is the case since all the vertices have the same out-degree and in-degree and they all behave in a similar manner. Therefore, changing the orientation of some edges would just change the direction of the information flow and not the intensity of the avalanche. Nevertheless, the lifetimes of avalanches are slightly longer in the directed network than on the undirected one. This suggests that it takes longer to transmit information between vertices in the directed circular network.

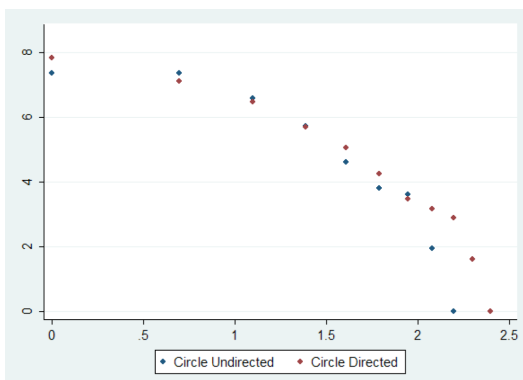
One important observation is that the distribution of avalanche sizes, areas and lifetimes on a log-log scale appears to be a straight line for the circular undirected and directed networks with degree of connectivity to the sink $w = 1$. We found that the fitted curves for the avalanche sizes for these graphs follow the same formula as the avalanche sizes for the grid in the previous section. The fitted curves are shown in Figure 5.11.



(a) Size

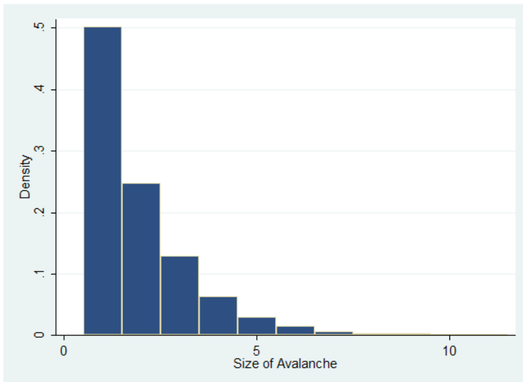


(b) Area

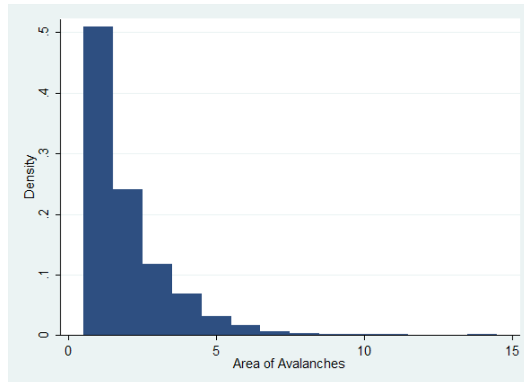


(c) Lifetime

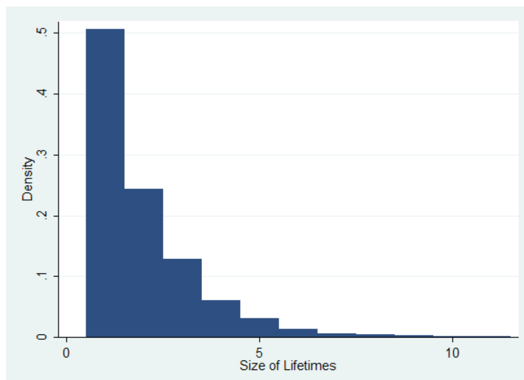
Figure 5.9: The distribution of avalanche strength (size, area, lifetime) on a directed circular graph compared to the one on an undirected circular graph on a log-log scale.



(a) Size

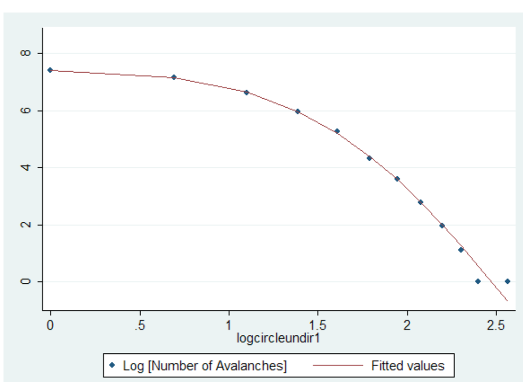


(b) Area

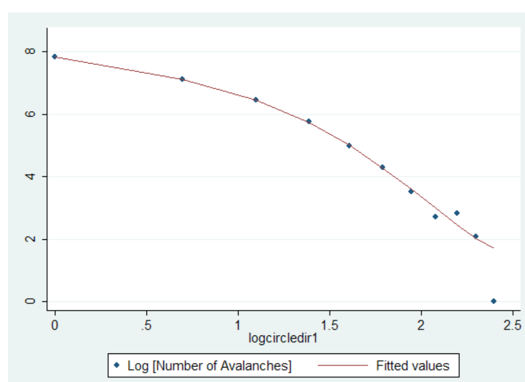


(c) Lifetime

Figure 5.10: The histograms of avalanche strength (size, area, lifetime) on a directed circular graph.



(a) Undirected

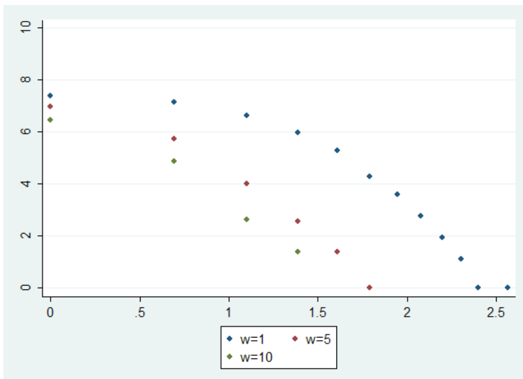


(b) Directed

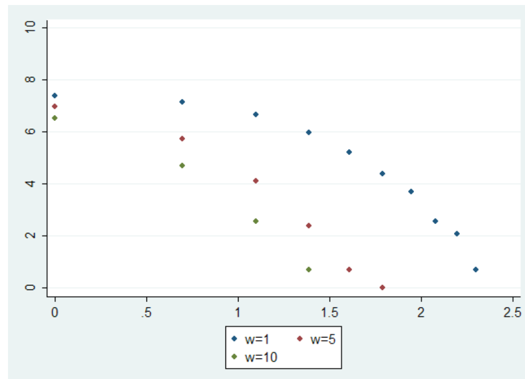
Figure 5.11: Fitted values for the avalanche sizes for the undirected and directed circular networks.

5.3.3 Changing the Degree of Connectivity to the Sink

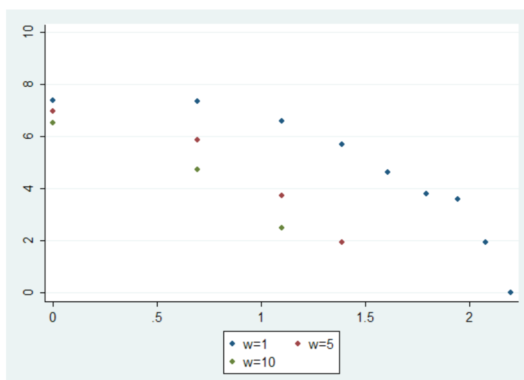
Since each vertex in the circular network is connected to only two other vertices and the sink, the degree of connectivity to the sink should be very important. Some experiments were done in which we changed the degree of connectivity to the sink from the initial value $w = 1$ to 5 and respectively 10. Different weights of the sink represent different degrees of information stickiness, a person's unwillingness to share information right away after receiving it. We conclude that, as w gets bigger, the size (and implicitly the area) and the lifetime of avalanches will decrease, as shown in Figure 5.12. This occurs since each vertex takes longer to fire as it retains each grain of sand for a longer period of time. Moreover, when taking high values of w, there were very few small avalanches. This implies that, as traders take longer time to pass on information, the market becomes more stable and the probability of an avalanche emerging decreases. We did not try to test whether the distribution on a log-log plot follows a straight line because of the limited number of observations.



(a) Size



(b) Area



(c) Lifetime

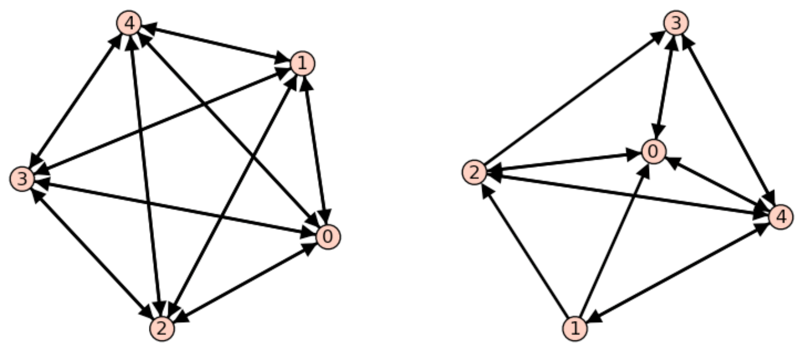
Figure 5.12: The distribution of avalanche sizes, areas and lifetimes on a circular graph (directed and undirected) with different weights of the sink (w) on a log-log scale.

The following experiments examine more types of networks that could exhibit a power-law behavior for the strengths of the avalanches. Since we do not get any power-law behavior in the following experiments, we concentrate on effects of changes in the avalanches dynamics in the networks due to changes in different parameters of the graphs. Although the analysis is based on the histograms of the avalanche strengths, the log-log plots of the distribution of these strengths are also included to show the departure from the power-law behavior.

5.4 Random Networks

Another network considered is the random network in which traders form connections with other traders at random and each connection has a "weight", representing the degree of influence traders have on other traders. This simulation considered a directed random graph with 100 vertices (plus sink) with a probability p that edges are formed between any two vertices and a maximum value for random edge weights denoted w. The actual weight of an edge e is a random number between 1 and $w(e)$ defined above. Two random graphs on 5 vertices with different specifications for p and w are shown in Figure 5.13.

Figures 5.14, 5.16, 5.18 and 5.20 show the distribution of avalanche sizes and areas for random graphs with different parameters p and w. The graph with $p = 1$ is a complete undirected graph with 101 vertices since every pair of distinct vertices is connected by an edge. This complete network corresponds to the case when every trader in the market is connected to all of the other traders. This way, the entire network could be considered a giant cluster. The histograms of sizes and areas of avalanches (Figures 5.15, 5.17, 5.19 and 5.21) suggest that for the case when both $p = 1$ and $w = 1$, most of the sizes and areas are close to either 0 or 100 since all of the vertices have the same degree of connectivity with each other and thus, they



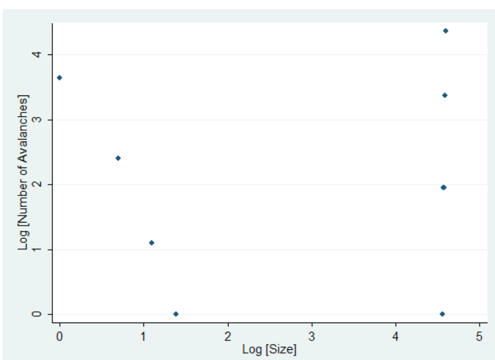
(a) p=1, w=1 (b) p=0.5, w=1

Figure 5.13: Random graphs on 5 vertices with different specifications.

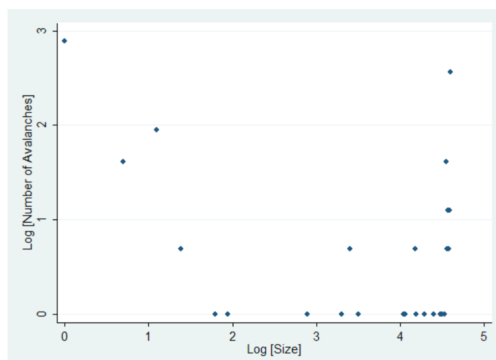
either all fire at once, or not fire at all. As we vary the range for the weight of edges (w) while keeping $p = 1$, we can see more variability for the areas and sizes. This also happens as we keep $w = 1$ but we vary the probability of edge connection (p).

One important characteristic of this network is the duration of avalanches. Figures 5.22 and 5.24 show that the power law behavior is not present anymore and that the lifetime of avalanches seem to be highly dependent on the parameters p and w. As we allow the weight of the edges to vary on a bigger range, there are also more chances of getting a higher weight. This higher weight would correspond to a longer "waiting time" (since it would take longer time for each vertex to fire). Moreover, upon the firing of a vertex a larger quantity of sand is lost so that the avalanches get shorter. This phenomenon is observed in the graphs of Figures 5.22 and 5.23 in which the avalanche duration is shown for the case when $p = 1$ and different values of w. The differences in the avalanche durations as w gets significantly bigger are almost negligible because of the random nature of w.

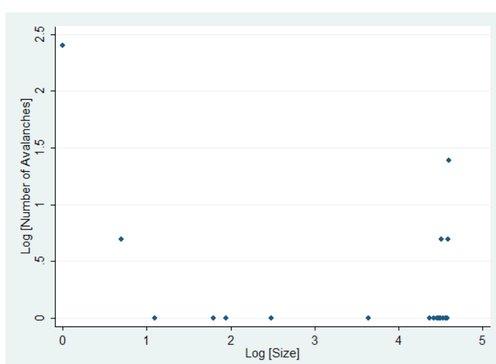
Since the weight assigned to each edge is a random positive number with an upper bound, we do not actually know anything about w unless we set it equal to 1. In this case, the weight of each edge is equal to 1, since the only random integer between 1 and 1 is 1. The avalanche lifetimes for the case when $w = 1$ and p varies are shown in Figures 5.24 and 5.25 and suggest that the duration of an avalanche is increasing by a lot as p decreases. This happens since a lower probability of edge connection implies fewer edges between vertices and thus the grains of sand would not reach too many vertices. Moreover, the duration of avalanches increases substantially as p decreases (from less than 10 to values that are above 100) for the same reason. The big difference between the case when $p = w = 1$ and all the other cases denotes the possibility of this being a critical state when traders are connected to all other traders and information flows freely between them. This rapid flow of information would causes sudden, large avalanches that can be compared to crashes in the stock market.



(a) p=1, w=1

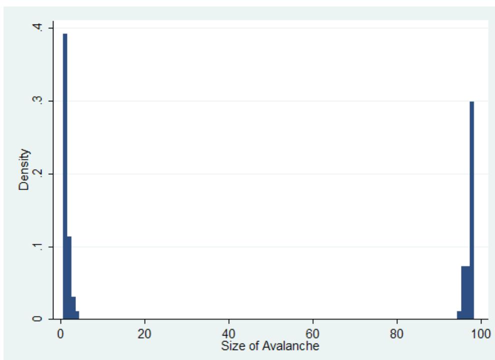


(b) p=1, w=3

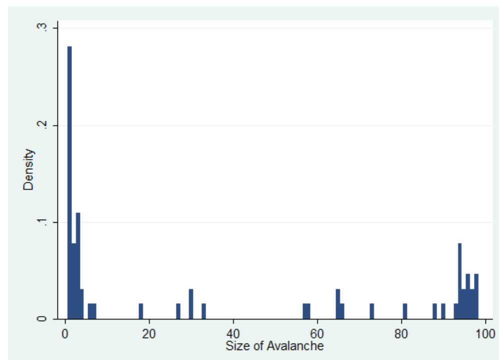


(c) p=1, w=10

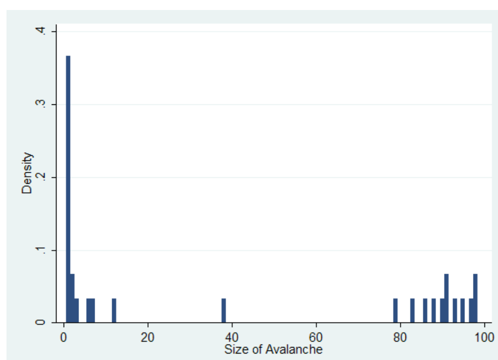
Figure 5.14: The distribution of avalanche sizes on a random graph for $p = 1$ (probability that edges occur) and w (maximum for random weight of edges).



(a) p=1, w=1

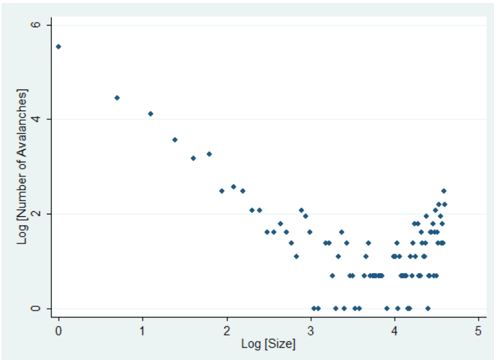


(b) p=1, w=3

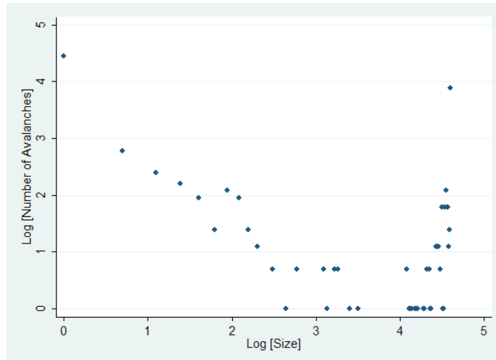


(c) p=1, w=10

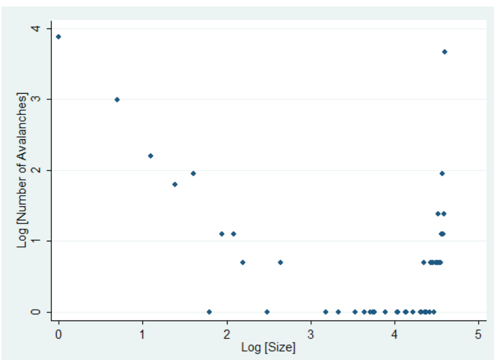
Figure 5.15: The histograms of avalanche sizes on a random graph for $p = 1$ (probability that edges occur) and w (maximum for random weight of edges).



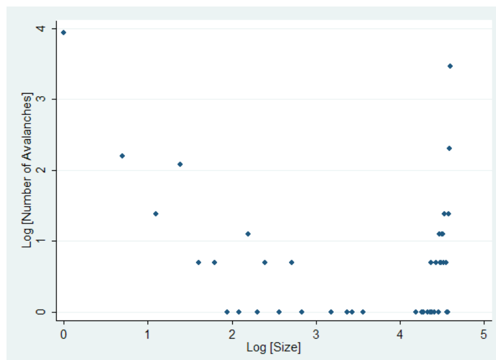
(a) p=0.15, w=1



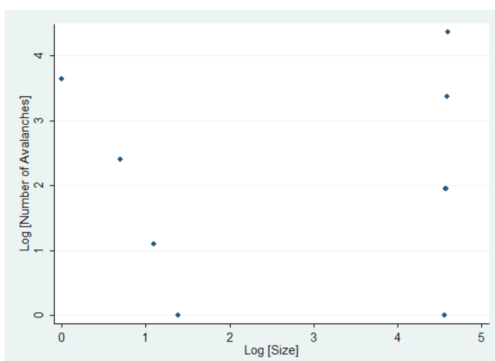
(b) p=0.5, w=1



(c) p=0.7, w=1

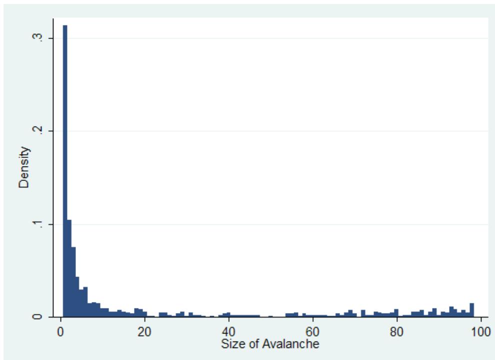


(d) p=0.9, w=1

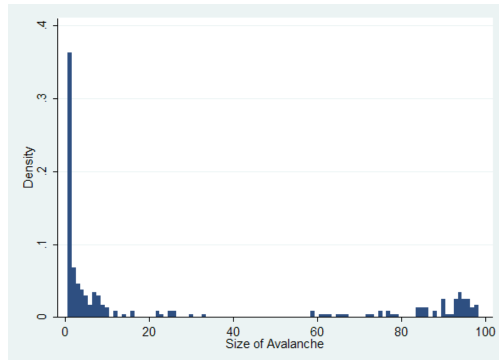


(e) p=1, w=1

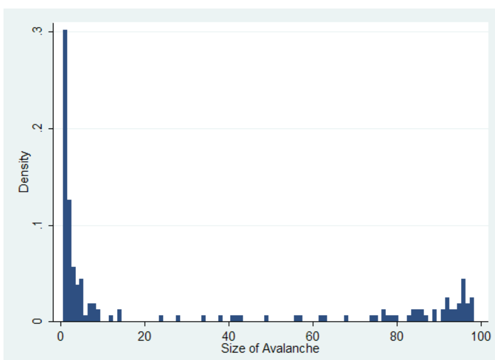
Figure 5.16: The distribution of avalanche sizes on a random graph for different values of p (probability that edges occur) and $w = 1$ (maximum for random weight of edges).



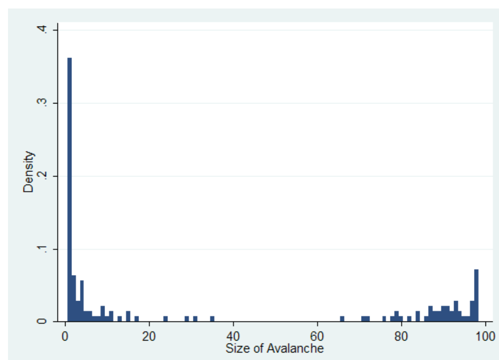
(a) p=0.15, w=1



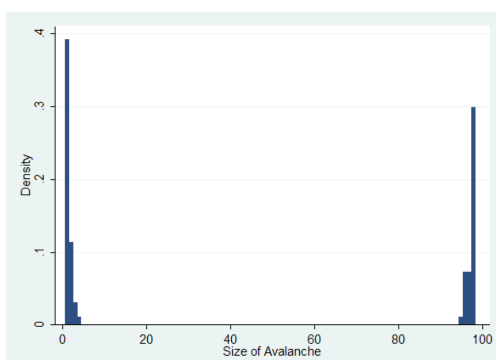
(b) p=0.5, w=1



(c) p=0.7, w=1

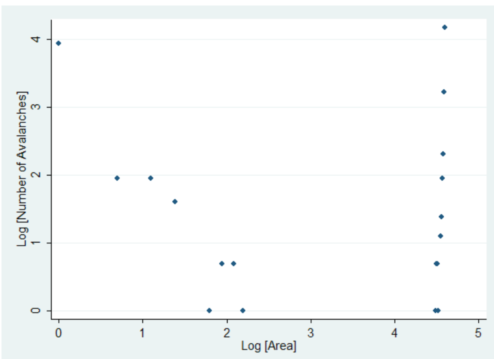


(d) p=0.9, w=1

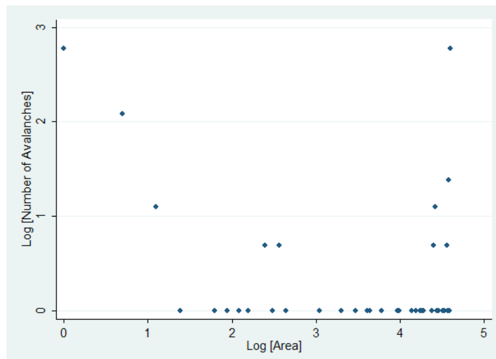


(e) p=1, w=1

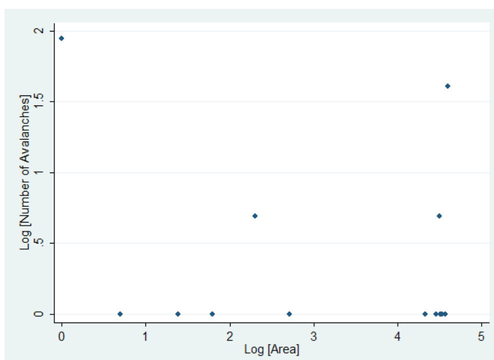
Figure 5.17: The histograms of avalanche sizes on a random graph for different values of p (probability that edges occur) and $w = 1$ (maximum for random weight of edges).



(a) p=1, w=1

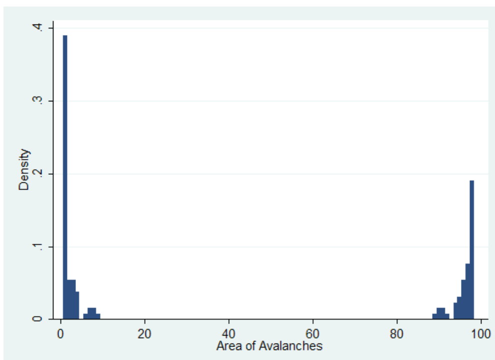


(b) p=1, w=3

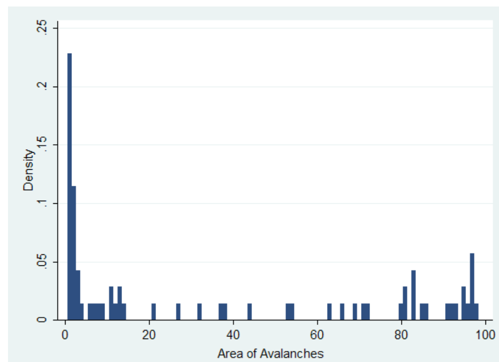


(c) p=1, w=10

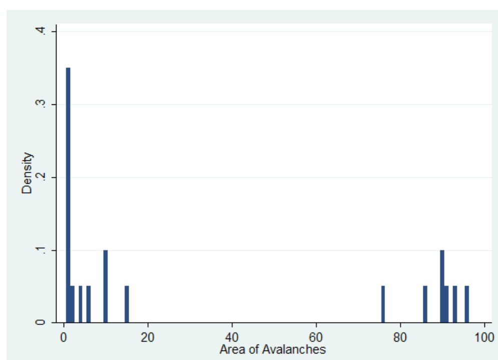
Figure 5.18: The distribution of avalanche areas on a random graph for different values of $p = 1$ (probability that edges occur) and w (maximum for random weight of edges).



(a) p=1, w=1

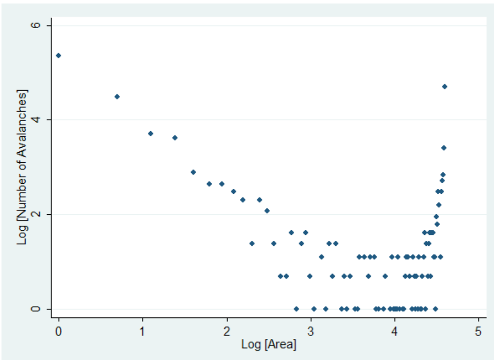


(b) p=1, w=3

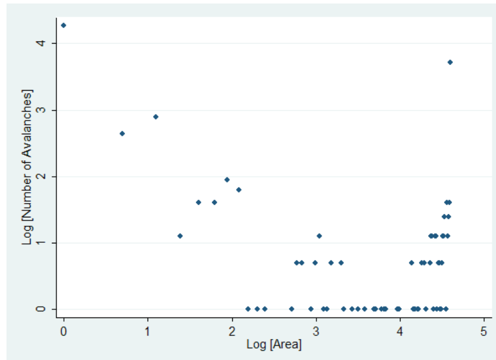


(c) p=1, w=10

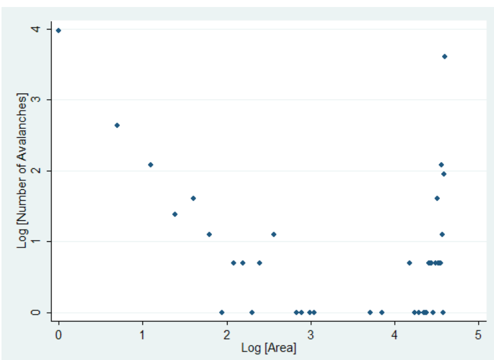
Figure 5.19: The histograms of avalanche areas on a random graph for different values of $p = 1$ (probability that edges occur) and w (maximum for random weight of edges).



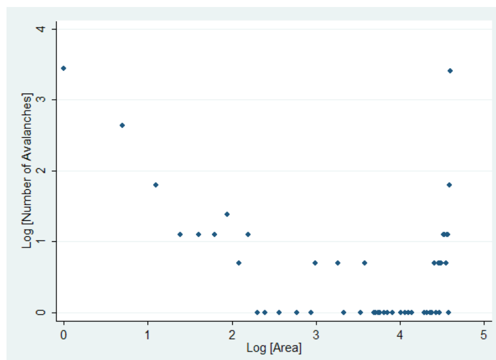
(a) p=0.15, w=1



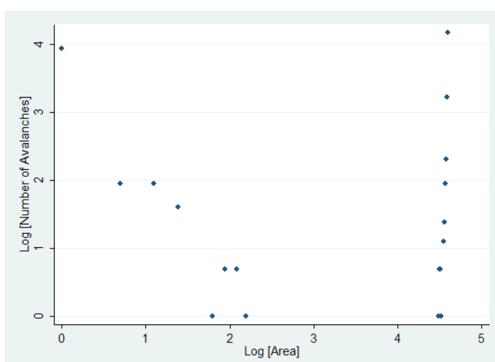
(b) p=0.5, w=1



(c) p=0.7, w=1

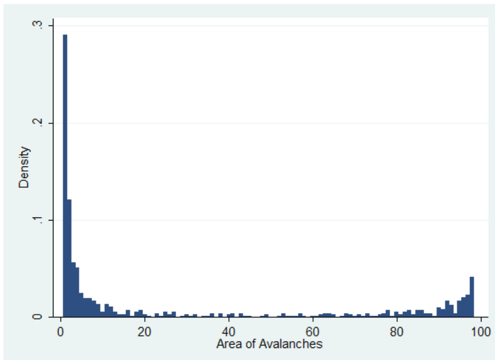


(d) p=0.9, w=1

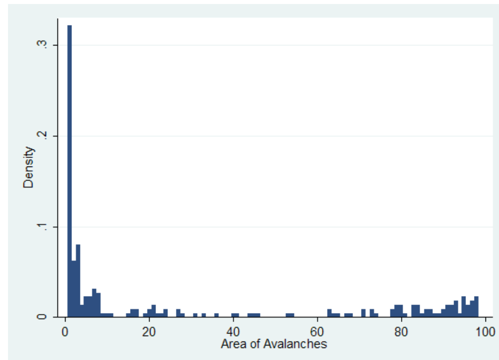


(e) p=1, w=1

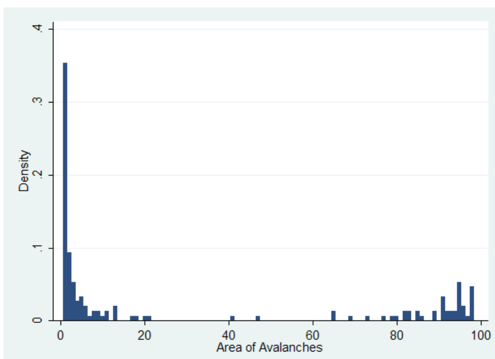
Figure 5.20: The distribution of avalanche areas on a random graph for different values of p (probability that edges occur) and $w = 1$ (maximum for random weight of edges).



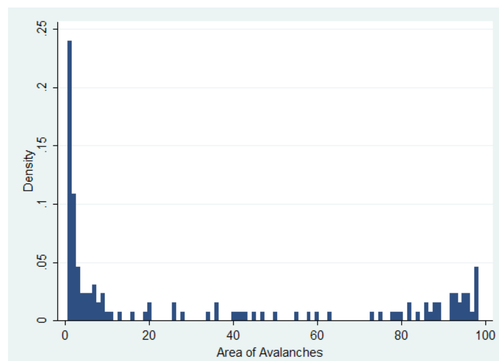
(a) p=0.15, w=1



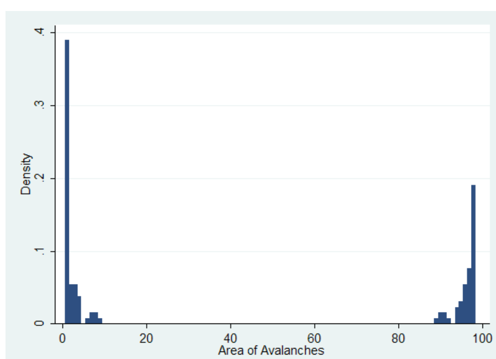
(b) p=0.5, w=1



(c) p=0.7, w=1

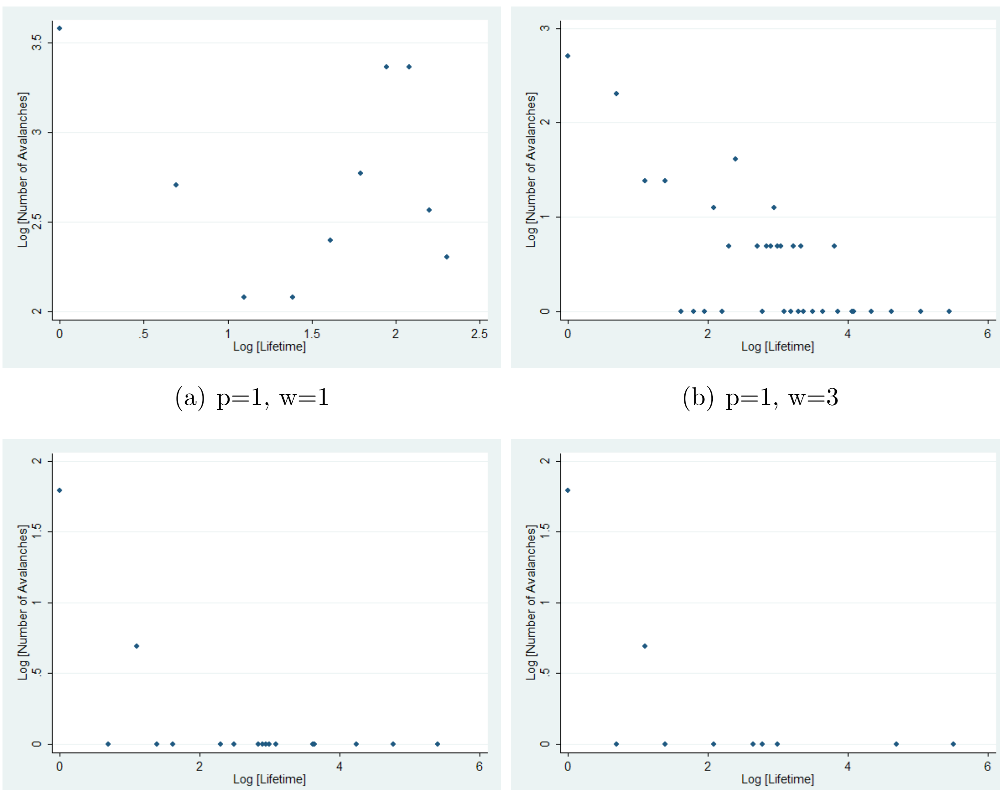


(d) p=0.9, w=1



(e) p=1, w=1

Figure 5.21: The histograms of avalanche areas on a random graph for different values of p (probability that edges occur) and $w = 1$ (maximum for random weight of edges).



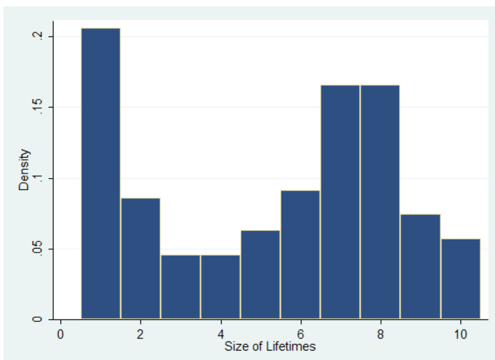
(a) p=1, w=1

(b) p=1, w=3

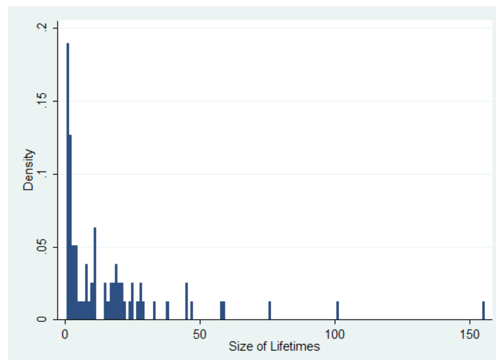
(c) p=1, w=10

(d) p=1, w=15

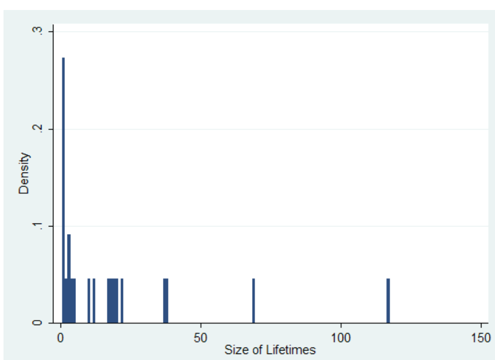
Figure 5.22: The distribution of avalanche lifetimes on a random graph for $p = 1$ (probability that edges occur) and different values for w (maximum for random weight of edges).



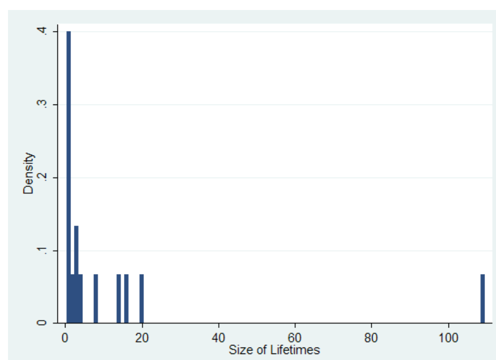
(a) p=1, w=1



(b) p=1, w=3

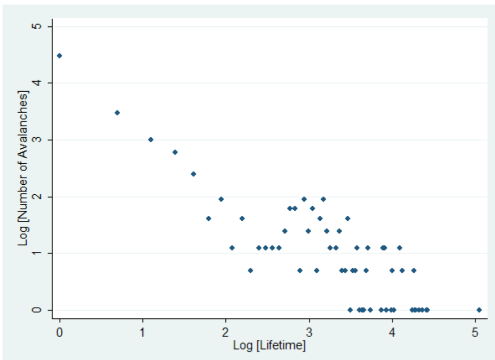


(c) p=1, w=10

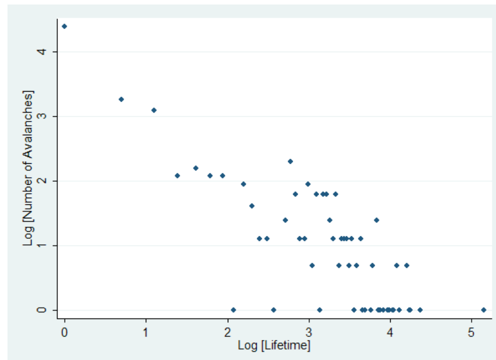


(d) p=1, w=15

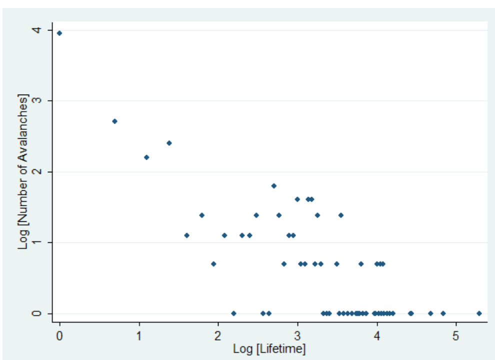
Figure 5.23: The histograms of avalanche lifetimes on a random graph for $p = 1$ (probability that edges occur) and different values for w (maximum for random weight of edges).



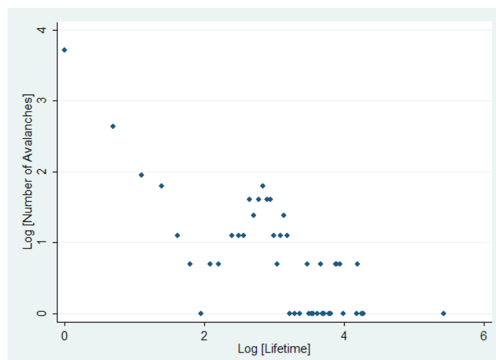
(a) p=0.4, w=1



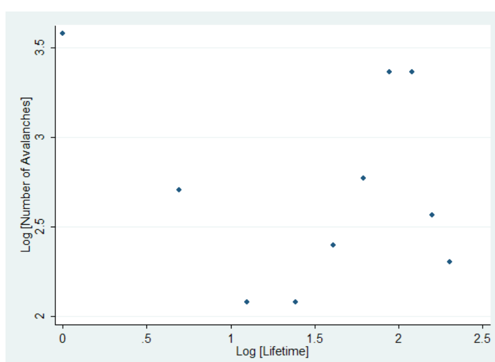
(b) p=0.5, w=1



(c) p=0.7, w=1

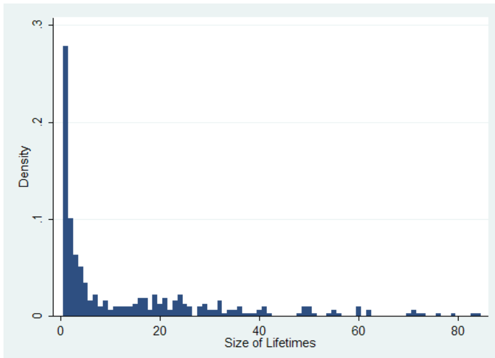


(d) p=0.9, w=1

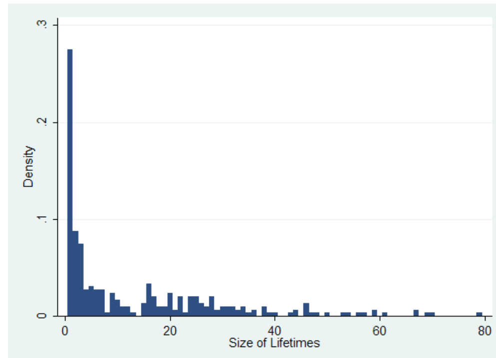


(e) p=1, w=1

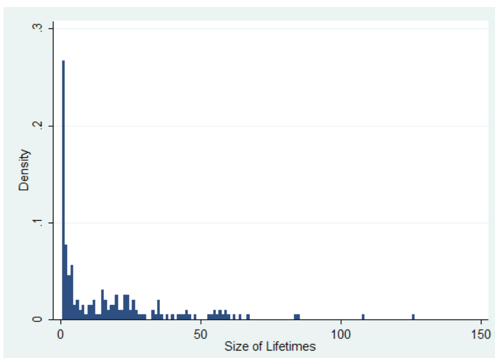
Figure 5.24: The distribution of avalanche lifetimes on a random graph for different values of p (probability that edges occur) and $w = 1$ (maximum for random weight of edges).



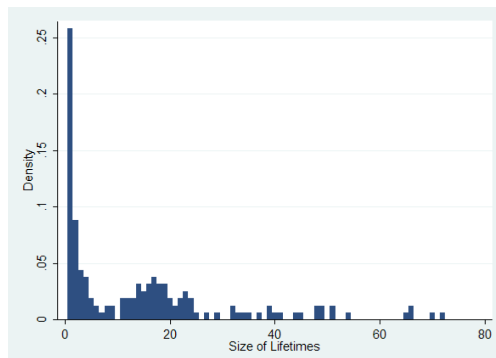
(a) p=0.4, w=1



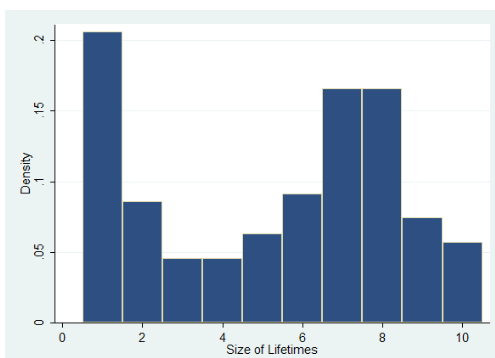
(b) p=0.5, w=1



(c) p=0.7, w=1



(d) p=0.9, w=1



(e) p=1, w=1

Figure 5.25: The histograms of avalanche lifetimes on a random graph for different values of p (probability that edges occur) and $w = 1$ (maximum for random weight of edges).

5.5 Network with a Special Node

We consider a random graph with 99 normal vertices, a special vertex and a sink attached. This graphs is defined by the parameters p, q and r, where p represents the probability that a pair of normal vertices are connected, q represents the probability that the normal vertices are connected to the sink and r is the probability that the special vertex is connected to a normal vertex, making sure that it is connected to at least one vertex. Also, the special vertex has in-degree 0, so that it cannot be influenced by any other vertex. An example of a random graph with a special vertex is shown in Figure 5.26 in which the special vertex is labeled “special”. This network tests the importance of having a vertex that has a different influence on the other vertices, most probably a bigger influence. This would be the case of a trader with a big influence on the population of traders. One problem with this experiment is that the random grain of sand that initiates the avalanches is dropped at random, and thus, we cannot analyze the influence of the special vertex in a very accurate way.

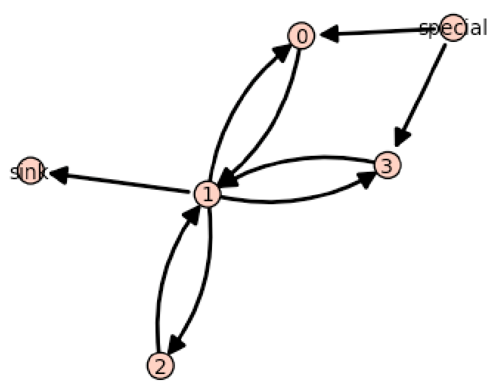


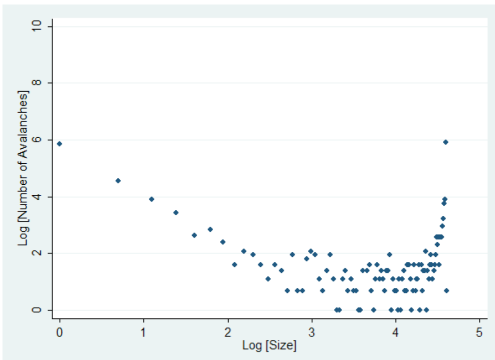
Figure 5.26: The random graph with a special vertex with $p = 0.3$ (probability that edges occur), $r = 0.3$ (probability of connection between the special vertex and the other vertices) and $q = 0.5$ (probability of sink connection).

In all of the experiments using this network we keep the connectivity to the sink constant at $q = 0.1$. At first, we examine the strength of avalanches while varying the probability that the special vertex is connected to a normal vertex. Thus, we conclude that the sizes and areas of avalanches (see Figures 5.27, 5.28, 5.29 and 5.30) are very close to either 0 or 100. This happens because of the very small connectivity of vertices. There are also very few changes in the sizes and areas as r varies since the only time this matters is when the random grain of sand is dropped at the special vertex. Figures 5.31 and 5.32 suggest that the durations of the avalanches in this case are very small, with very few big ones.

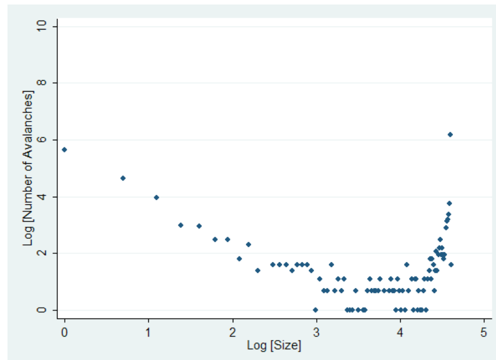
In the case that $p = r$, the special vertex can be treated as a normal vertex (except for the fact that its out-degree is 0) and we get similar results for the areas and sizes of avalanches a in the previous case (see Figures 5.33, 5.34, 5.35 and 5.36). The duration of avalanches is again very small (smaller than 50 in general), with few large ones. As p and r get small, there is a very big avalanche which takes place when the grain of sand is dropped at a vertex that forms part of a cluster.[7]

The following network investigates further the transmission of information in a graph with clusters.

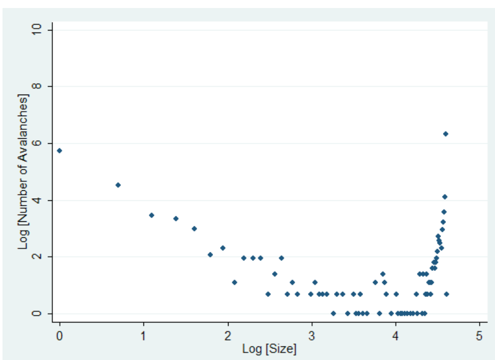
[7]Since there the probability of an edge between two vertices I very small, clusters are being formed.



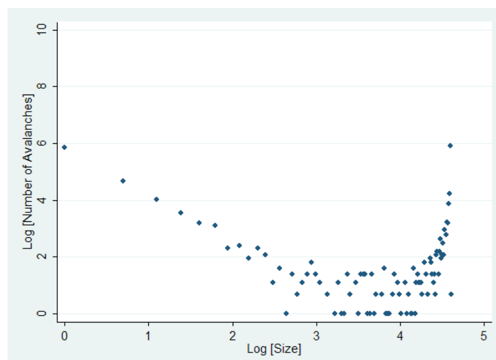
(a) p=0.1, q=0.1, r=0.1



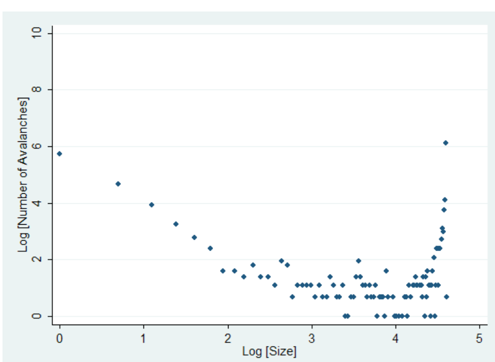
(b) p=0.1, q=0.1, r=0.2



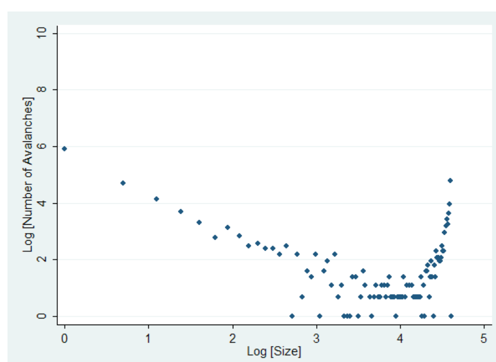
(c) p=0.1, q=0.1, r=0.4



(d) p=0.1, q=0.1, r=0.6

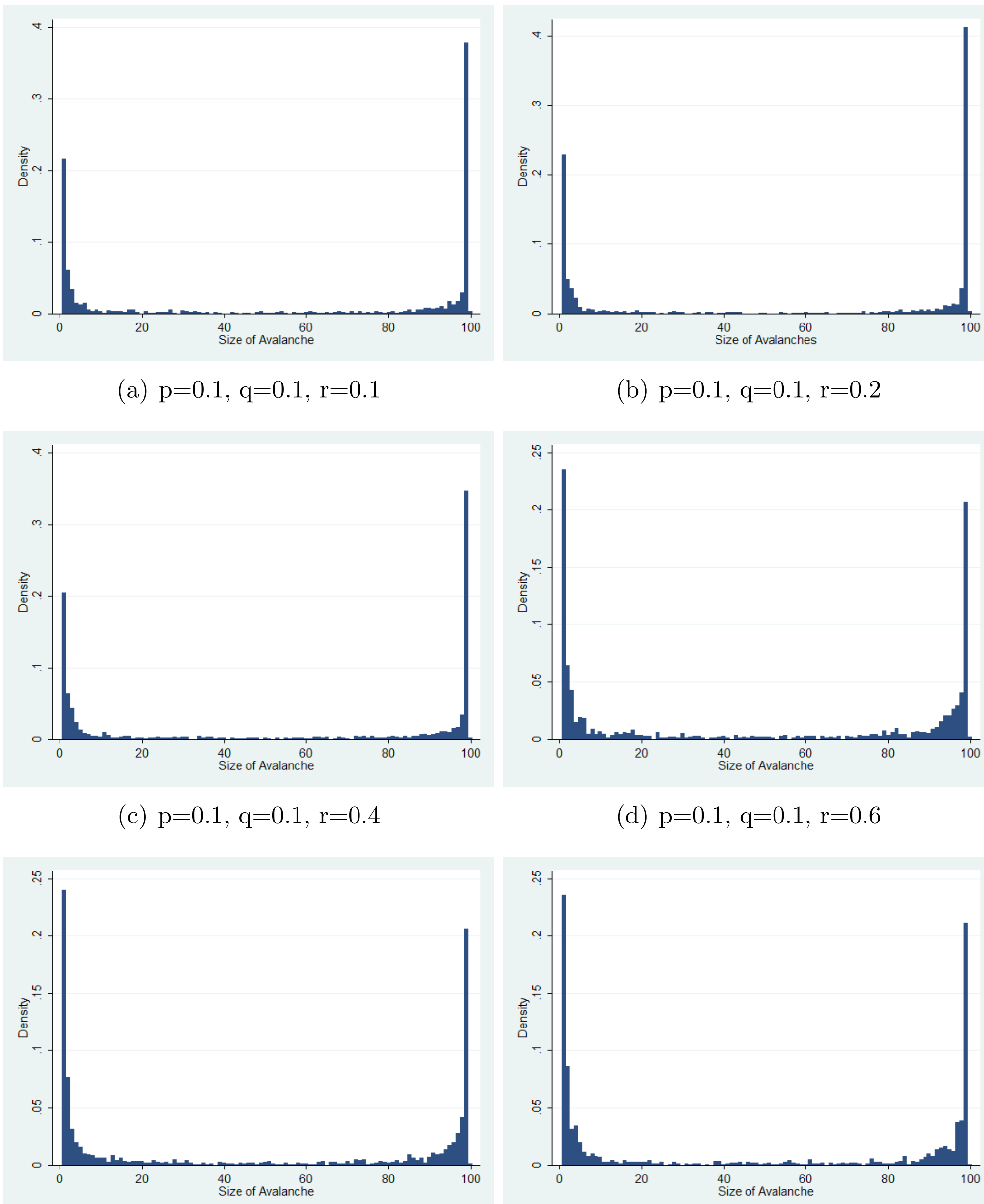


(e) p=0.1, q=0.1, r=0.8



(f) p=0.1, q=0.1, r=1

Figure 5.27: The distribution of avalanche sizes on a random graph for different values q (probability of sink connection), $p = 0.1$ (probability that edges occur) and $r = 0.1$ (probability of connection between the special vertex and the other vertices).



(a) p=0.1, q=0.1, r=0.1

(b) p=0.1, q=0.1, r=0.2

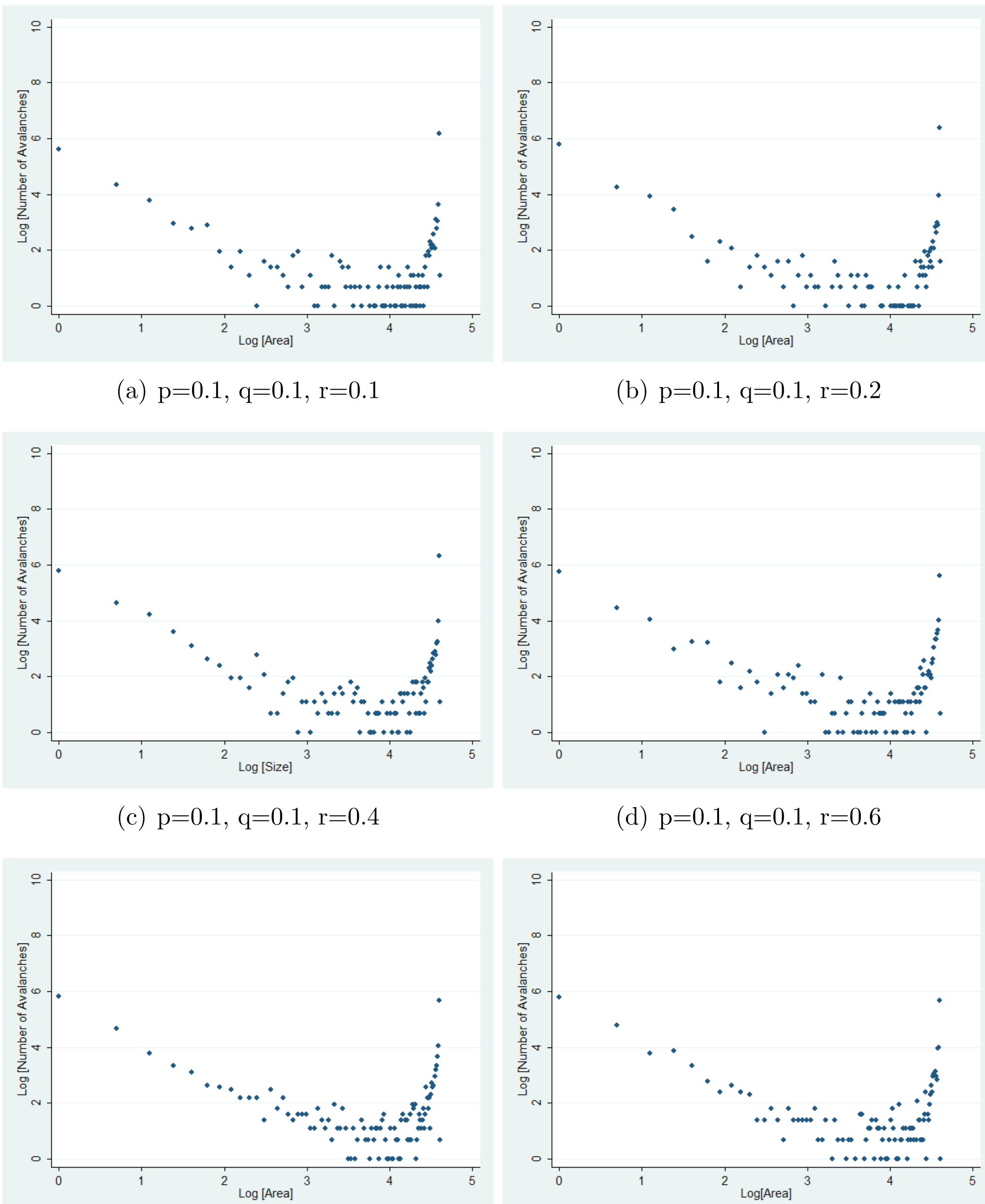
(c) p=0.1, q=0.1, r=0.4

(d) p=0.1, q=0.1, r=0.6

(e) p=0.1, q=0.1, r=0.8

(f) p=0.1, q=0.1, r=1

Figure 5.28: The histograms of avalanche sizes on a random graph for different values q (probability of sink connection), $p = 0.1$ (probability that edges occur) and $r = 0.1$ (probability of connection between the special vertex and the other vertices).



(a) p=0.1, q=0.1, r=0.1

(b) p=0.1, q=0.1, r=0.2

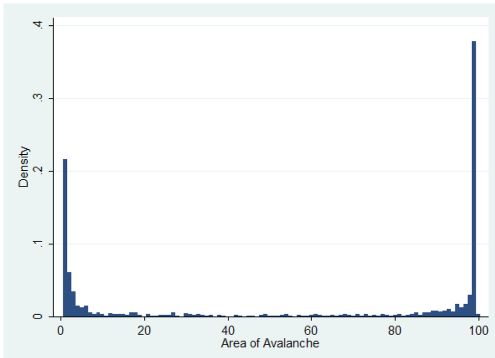
(c) p=0.1, q=0.1, r=0.4

(d) p=0.1, q=0.1, r=0.6

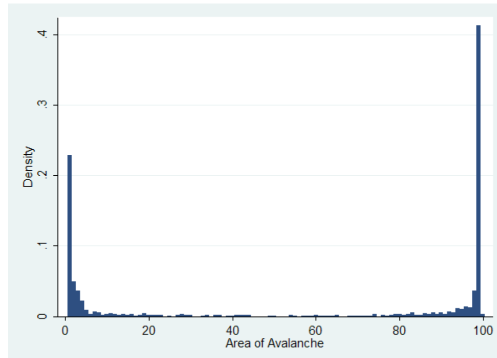
(e) p=0.1, q=0.1, r=0.8

(f) p=0.1, q=0.1, r=1

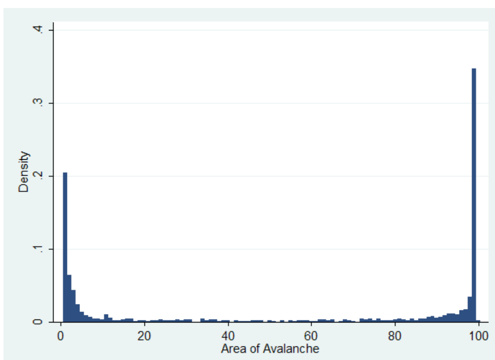
Figure 5.29: The distribution of avalanche areas on a random graph for different values q (probability of sink connection), $p = 0.1$ (probability that edges occur) and $r = 0.1$ (probability of connection between the special vertex and the other vertices).



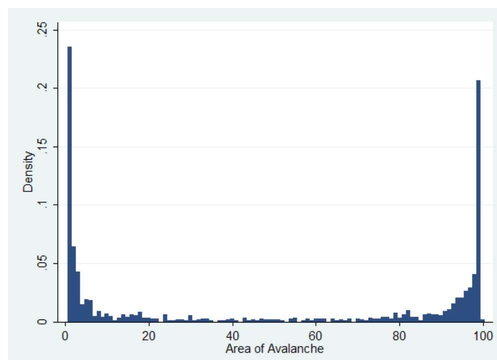
(a) p=0.1, q=0.1, r=0.1



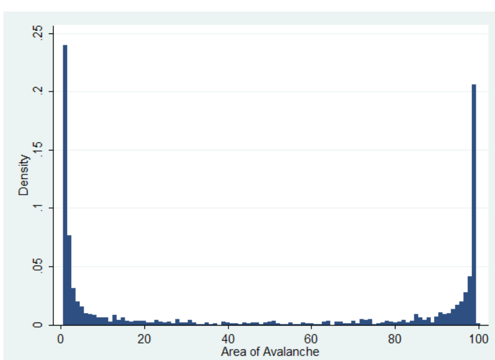
(b) p=0.1, q=0.1, r=0.2



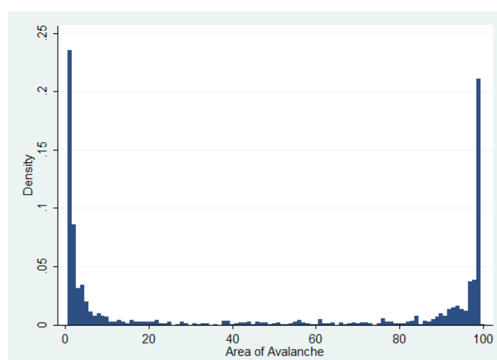
(c) p=0.1, q=0.1, r=0.4



(d) p=0.1, q=0.1, r=0.6

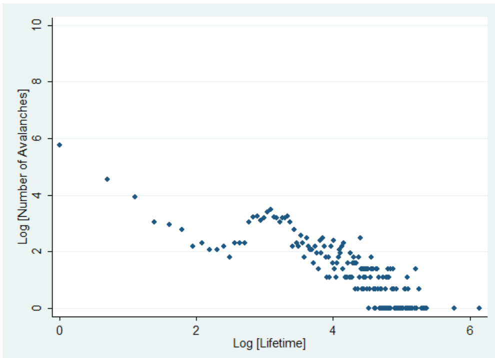


(e) p=0.1, q=0.1, r=0.8

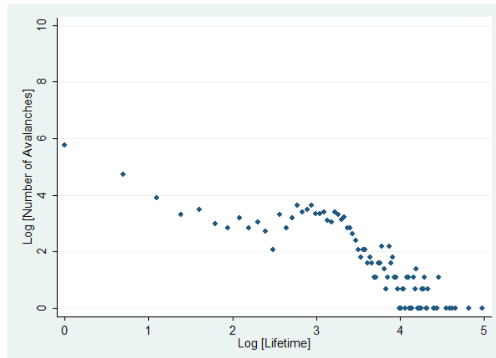


(f) p=0.1, q=0.1, r=1

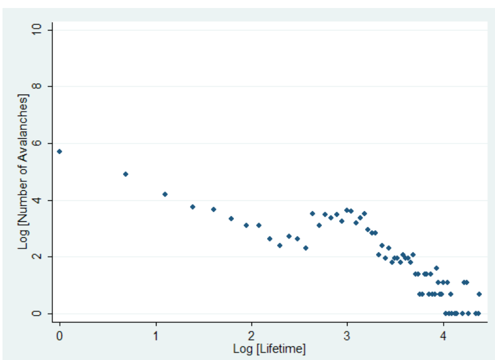
Figure 5.30: The histograms of avalanche areas on a random graph for different values q (probability of sink connection), $p = 0.1$ (probability that edges occur) and $r = 0.1$ (probability of connection between the special vertex and the other vertices).



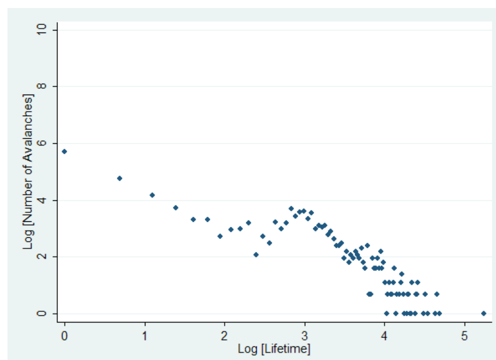
(a) p=0.1, q=0.1, r=0.1



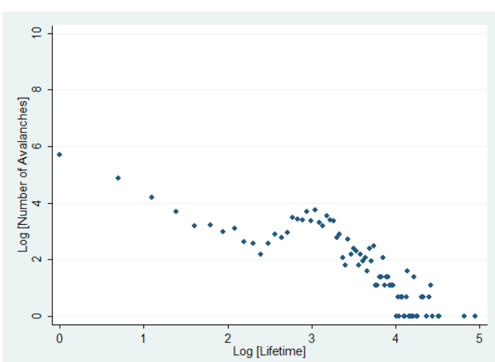
(b) p=0.1, q=0.1, r=0.2



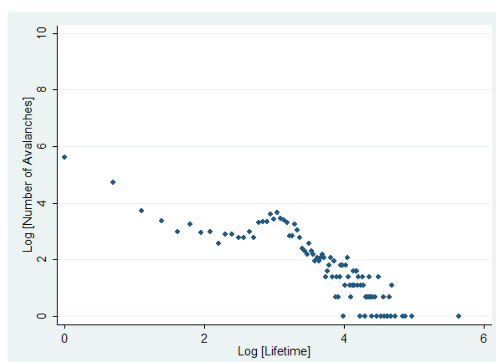
(c) p=0.1, q=0.1, r=0.4



(d) p=0.1, q=0.1, r=0.6

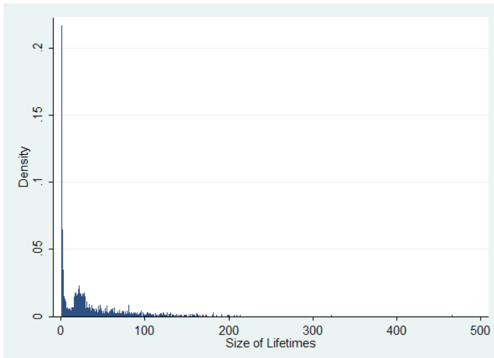


(e) p=0.1, q=0.1, r=0.8

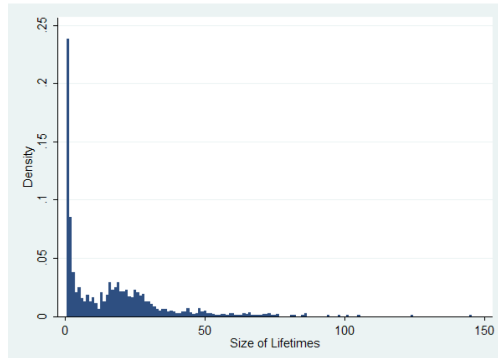


(f) p=0.1, q=0.1, r=1

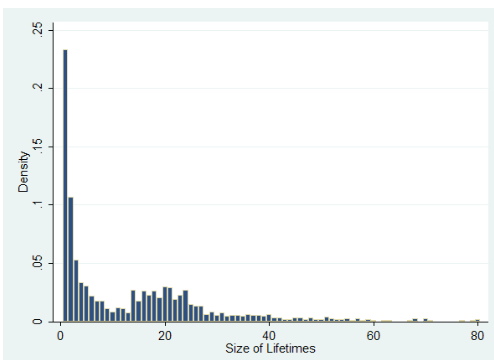
Figure 5.31: The distribution of avalanche lifetimes on a random graph for different values q (probability of sink connection), $p = 0.1$ (probability that edges occur) and $r = 0.1$ (probability of connection between the special vertex and the other vertices).



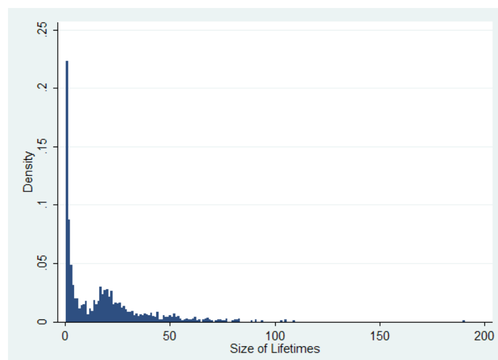
(a) p=0.1, q=0.1, r=0.1



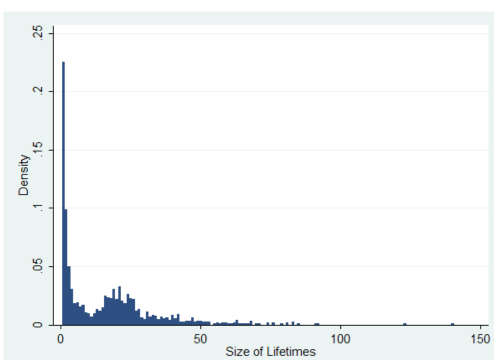
(b) p=0.1, q=0.1, r=0.2



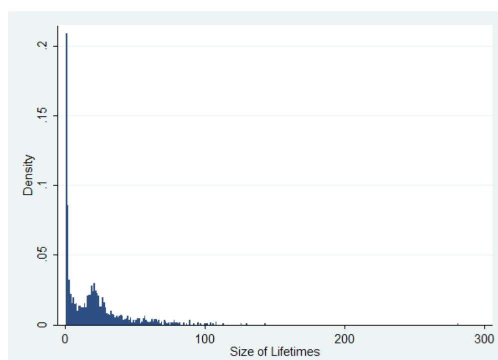
(c) p=0.1, q=0.1, r=0.4



(d) p=0.1, q=0.1, r=0.6

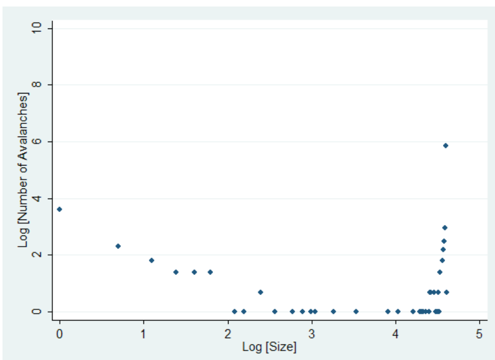


(e) p=0.1, q=0.1, r=0.8

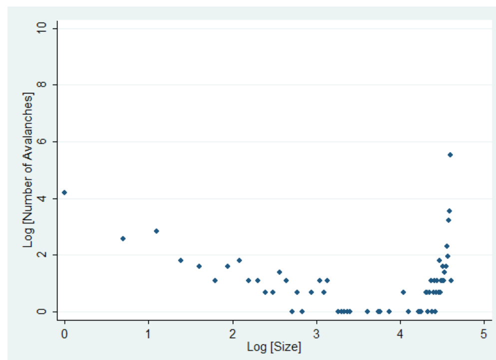


(f) p=0.1, q=0.1, r=1

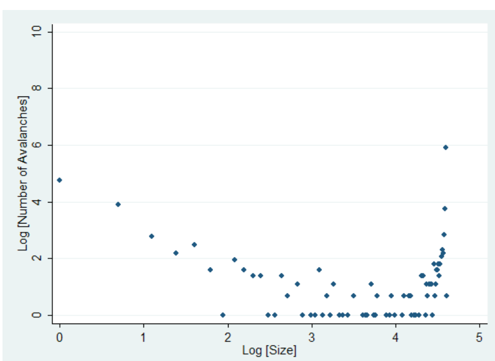
Figure 5.32: The histograms of avalanche lifetimes on a random graph for different values q (probability of sink connection), $p = 0.1$ (probability that edges occur) and $r = 0.1$ (probability of connection between the special vertex and the other vertices).



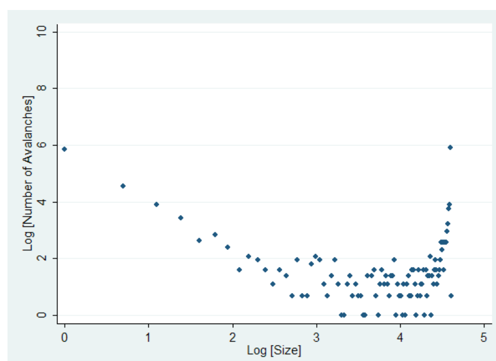
(a) p=0.4, q=0.1, r=0.4



(b) p=0.3, q=0.1, r=0.3

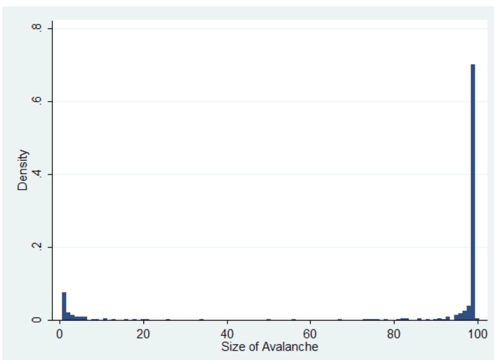


(c) p=0.2, q=0.1, r=0.2

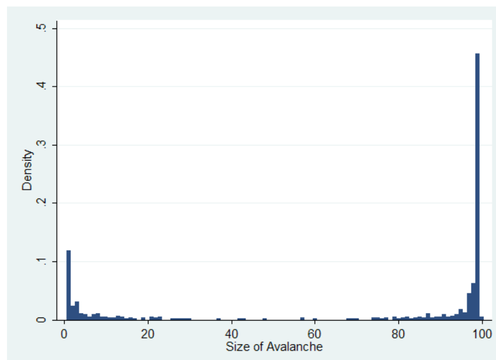


(d) p=0.1, q=0.1, r=0.1

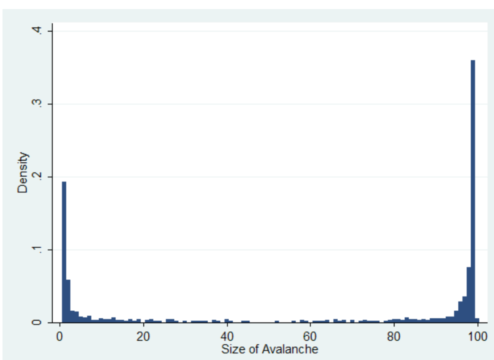
Figure 5.33: The distribution of avalanche sizes on a random graph for different values of p (probability that edges occur), r (probability of connection between the special vertex and the other vertices) and $q = 0.1$ (probability of sink connection).



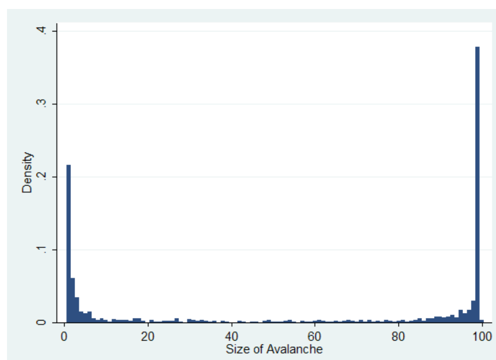
(a) p=0.4, q=0.1, r=0.4



(b) p=0.3, q=0.1, r=0.3

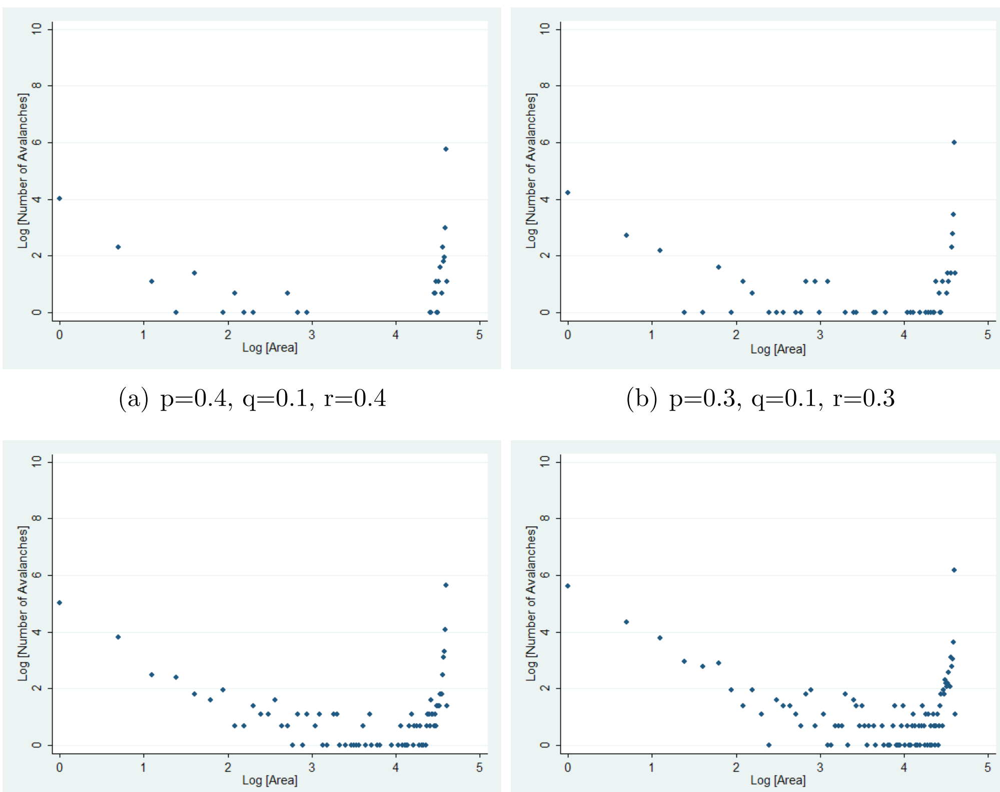


(c) p=0.2, q=0.1, r=0.2



(d) p=0.1, q=0.1, r=0.1

Figure 5.34: The histograms of avalanche sizes on a random graph for different values of p (probability that edges occur), r (probability of connection between the special vertex and the other vertices) and $q = 0.1$ (probability of sink connection).



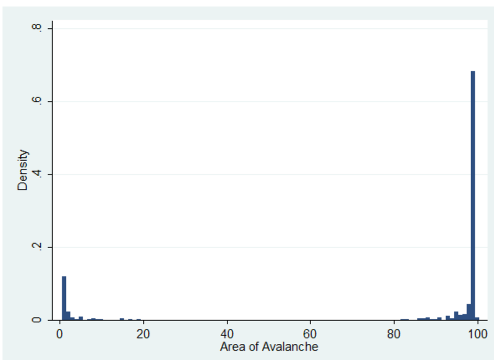
(a) p=0.4, q=0.1, r=0.4

(b) p=0.3, q=0.1, r=0.3

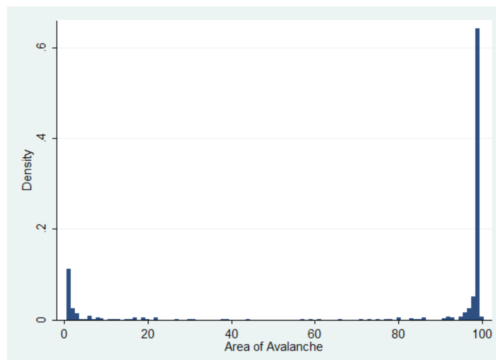
(c) p=0.2, q=0.1, r=0.2

(d) p=0.1, q=0.1, r=0.1

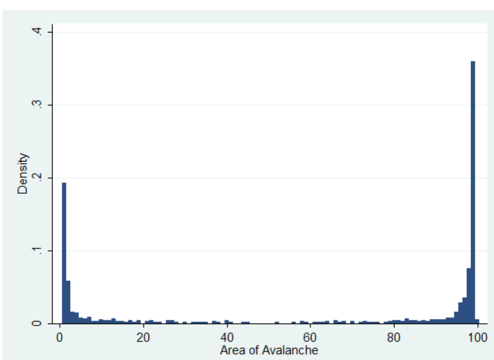
Figure 5.35: The distribution of avalanche areas on a random graph for different values of p (probability that edges occur), r (probability of connection between the special vertex and the other vertices) and $q = 0.1$ (probability of sink connection).



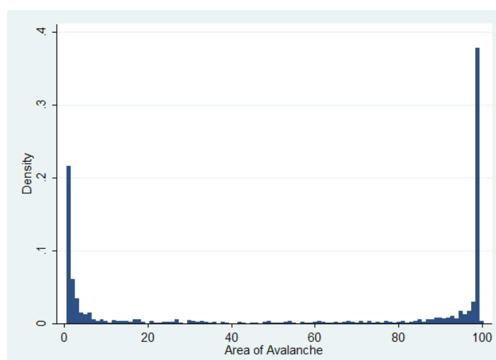
(a) p=0.4, q=0.1, r=0.4



(b) p=0.3, q=0.1, r=0.3

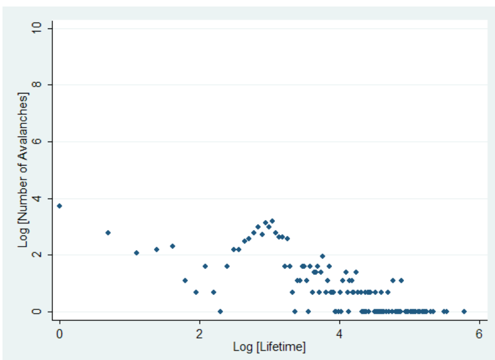


(c) p=0.2, q=0.1, r=0.2

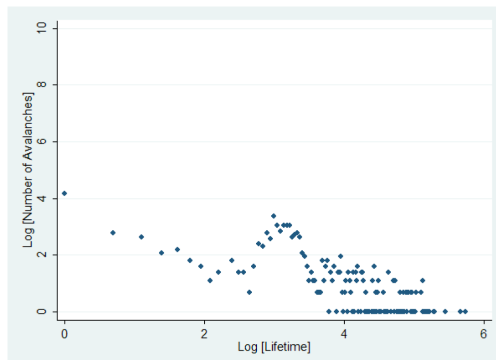


(d) p=0.1, q=0.1, r=0.1

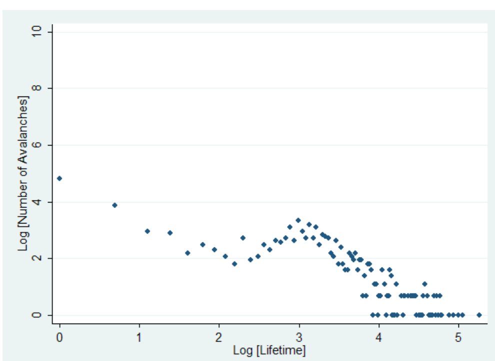
Figure 5.36: The histograms of avalanche areas on a random graph for different values of p (probability that edges occur), r (probability of connection between the special vertex and the other vertices) and $q = 0.1$ (probability of sink connection).



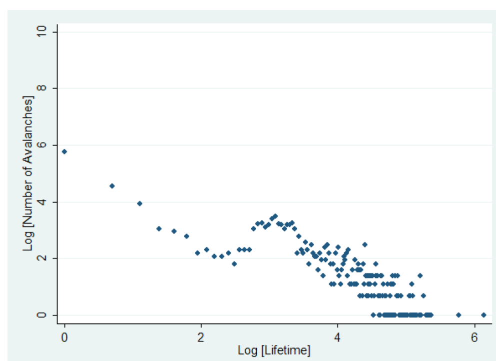
(a) p=0.4, q=0.1, r=0.4



(b) p=0.3, q=0.1, r=0.3

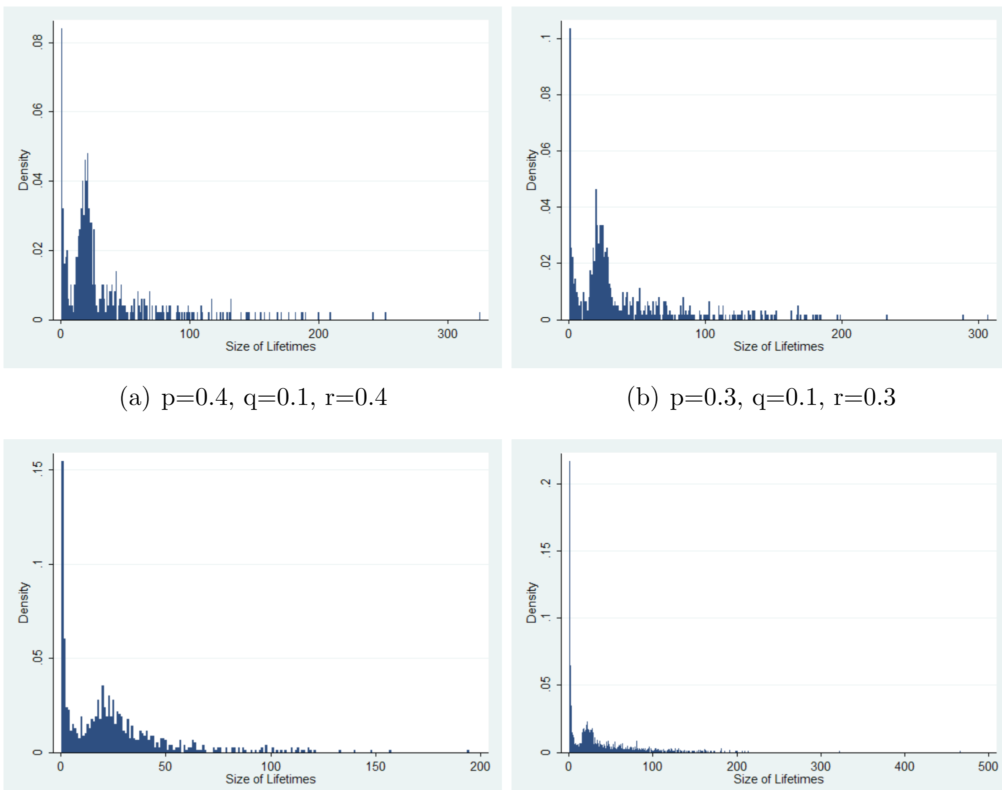


(c) p=0.2, q=0.1, r=0.2



(d) p=0.1, q=0.1, r=0.1

Figure 5.37: The distribution of avalanche lifetimes on a random graph for different values of p (probability that edges occur), r (probability of connection between the special vertex and the other vertices) and $q = 0.1$ (probability of sink connection).



(a) p=0.4, q=0.1, r=0.4

(b) p=0.3, q=0.1, r=0.3

(c) p=0.2, q=0.1, r=0.2

(d) p=0.1, q=0.1, r=0.1

Figure 5.38: The histograms of avalanche lifetimes on a random graph for different values of p (probability that edges occur), r (probability of connection between the special vertex and the other vertices) and $q = 0.1$ (probability of sink connection).

5.6 Network with Clusters

According to Srinivasa (2006), random graphs are not the best way of modeling a communication network since they assume that the probability of a person being connected to a friend is the same as being connected to a person from a different country. Furthermore, he suggests that a better approximation of information network would be a clustered graph consisting of different clusters occasionally connected among each other. This way, we constructed a graph with a total of 100 vertices formed by taking m different complete graphs connected by an edge at random. Each vertex of these complete graphs has probability p of being connected to the sink. Also, the graph is constructed in such a way that at least one vertex in each cluster is attached to the sink. Figure in 5.39.

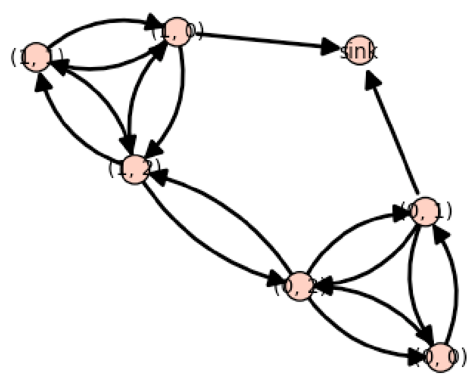
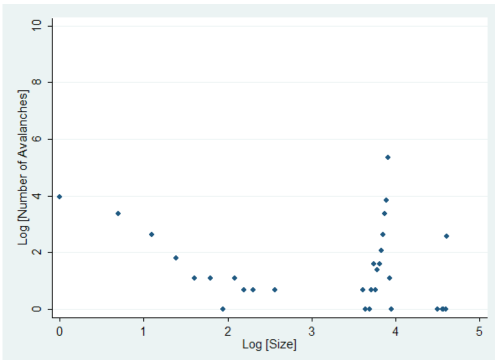


Figure 5.39: The clustered graph with two clusters consisting of 3 vertices each and probability of sink connection $p = 0.1$.

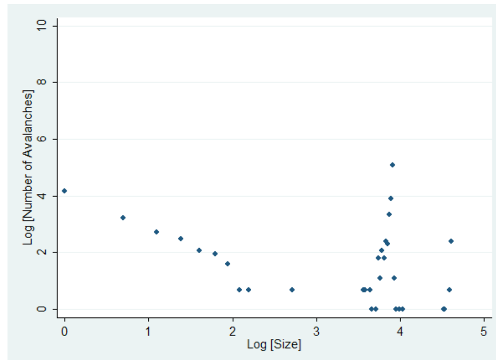
Three different networks were considered in this experiment, each with 100 vertices and one sink vertex. The first graph consisted of two clusters, each with 50 vertices, connected with each other by a random edge and connected to the sink with probability p. In this case, the sizes and areas of avalanches (see Figures 5.40, 5.41, 5.42 and 5.43) were mostly clustered around 0 and 50, with few clustered around 100.

This suggests that since the random grain of sand is dropped in one of the clusters, the transmission of information is highly effective inside that cluster, but it seldom reaches the other cluster. This phenomenon can be seen in the other two networks considered, one with five clusters with 20 vertices each, and one with two clusters of 30 and respectively 70 vertices. The distributions of sizes and areas for these networks can be seen in Figures 5.46, 5.47, 5.48 and 5.49 (for the first network) and Figures 5.52, 5.53, 5.54 and 5.55 (for the second network).

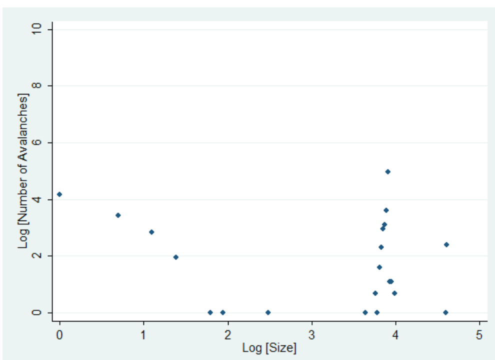
We can also see that the lifetimes of the avalanches get smaller as the probability of sink connection (p) increases. This makes sense since the more edges are connected to the sink, the more grains of sand are lost and so the less time needed for each vertex to become stable. This phenomenon was observed for all three different networks (see Figures 5.44, 5.45, 5.50, 5.51, 5.56 and 5.57).



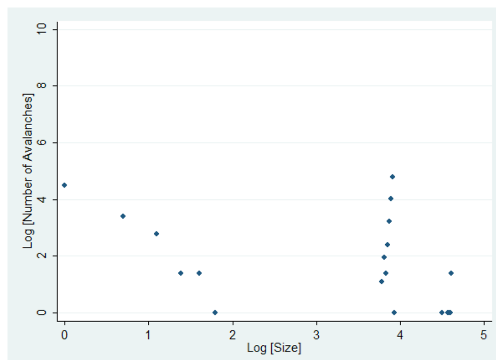
(a) p=0.2



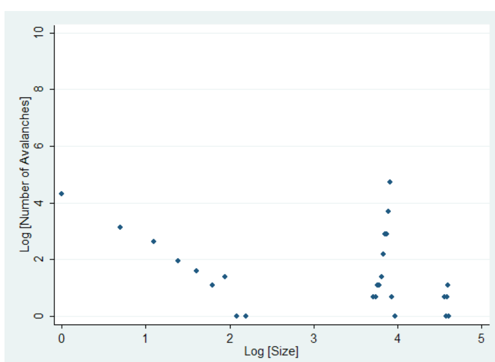
(b) p=0.5



(c) p=0.7

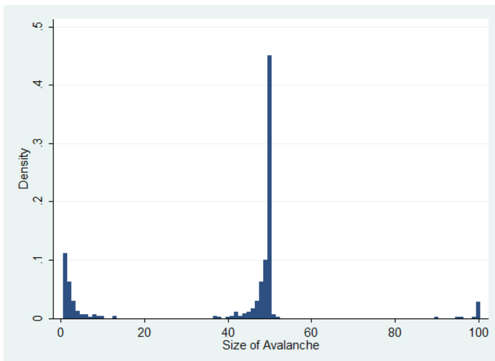


(d) p=0.9

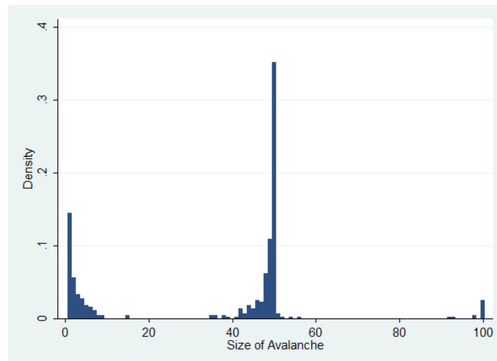


(e) p=1

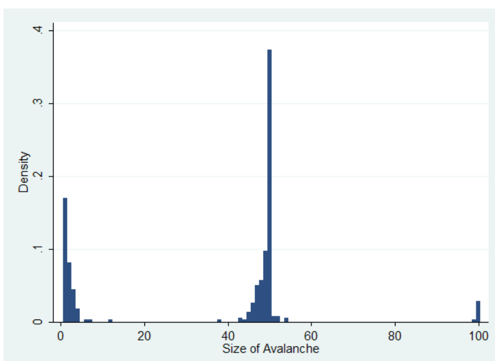
Figure 5.40: The distribution of avalanche sizes on a clustered graph with two clusters with 50 vertices for different values of p (probability of sink connection).



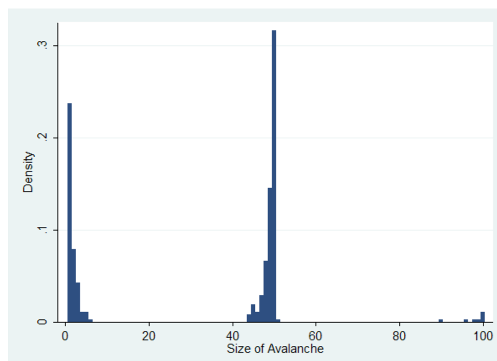
(a) p=0.2



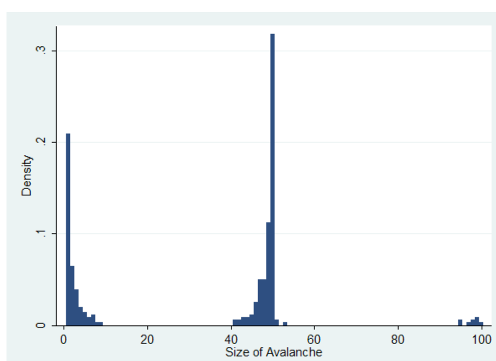
(b) p=0.5



(c) p=0.7

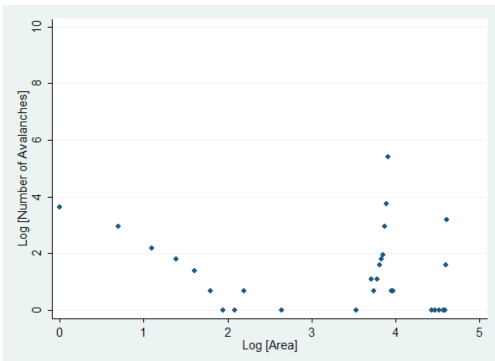


(d) p=0.9

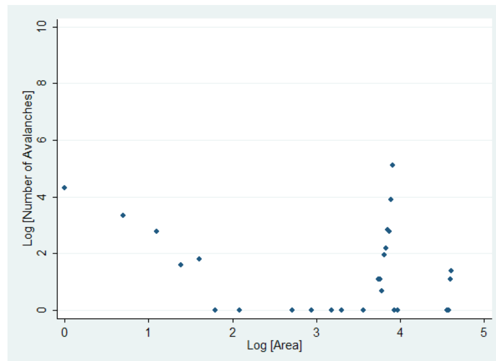


(e) p=1

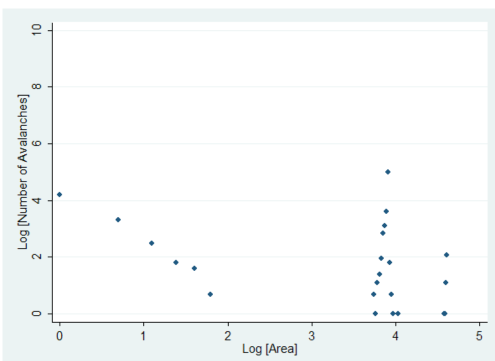
Figure 5.41: The histograms of avalanche sizes on a clustered graph with two clusters with 50 vertices for different values of p (probability of sink connection).



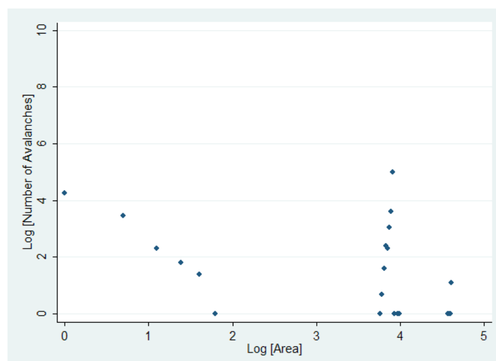
(a) p=0.2



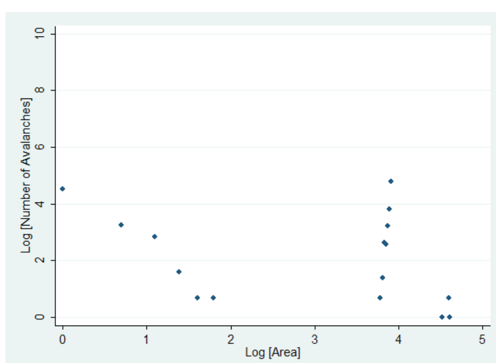
(b) p=0.5



(c) p=0.7

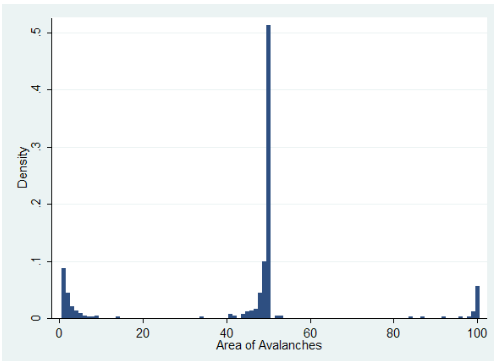


(d) p=0.9

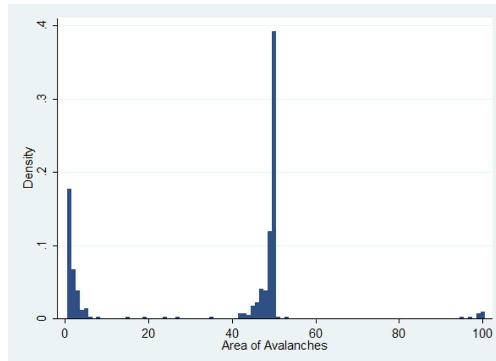


(e) p=1

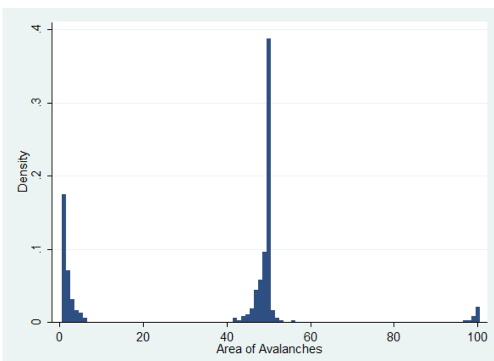
Figure 5.42: The distribution of avalanche areas on a clustered graph with two clusters with 50 vertices for different values of p (probability of sink connection).



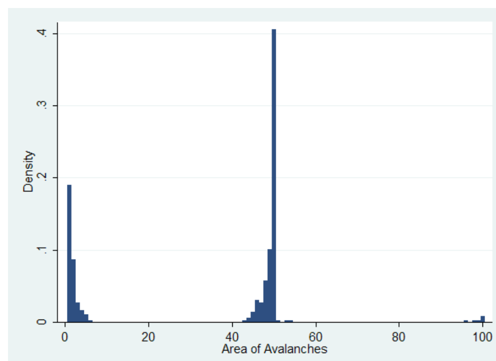
(a) p=0.2



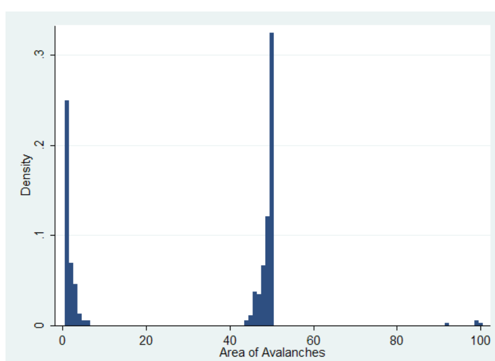
(b) p=0.5



(c) p=0.7

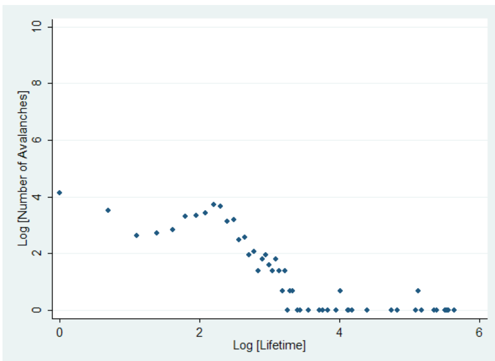


(d) p=0.9

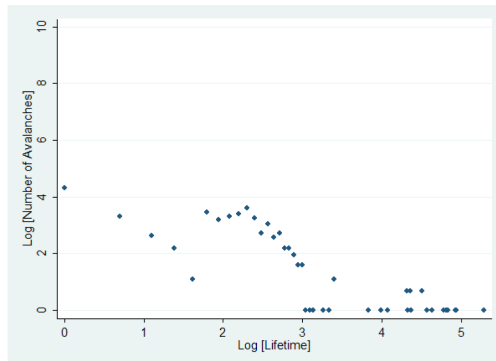


(e) p=1

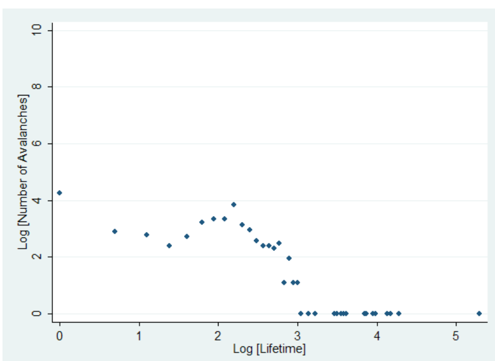
Figure 5.43: The histograms of avalanche areas on a clustered graph with two clusters with 50 vertices for different values of p (probability of sink connection).



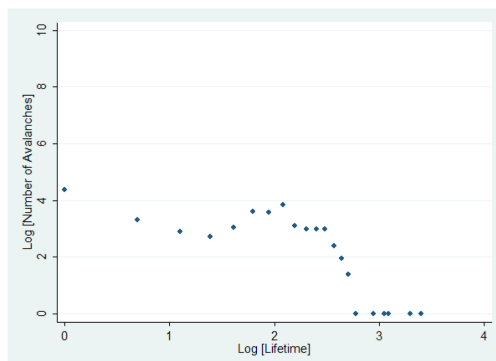
(a) p=0.2



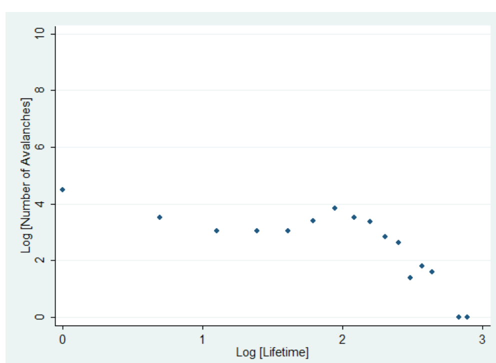
(b) p=0.5



(c) p=0.7

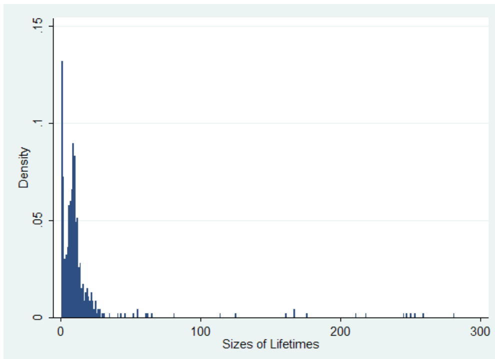


(d) p=0.9

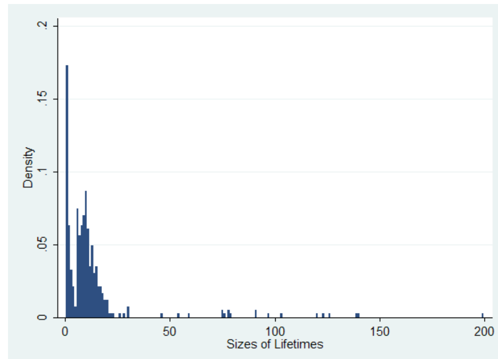


(e) p=1

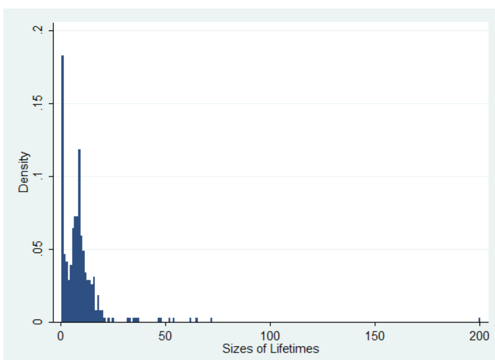
Figure 5.44: The distribution of avalanche lifetimes on a clustered graph with two clusters with 50 vertices for different values of p (probability of sink connection).



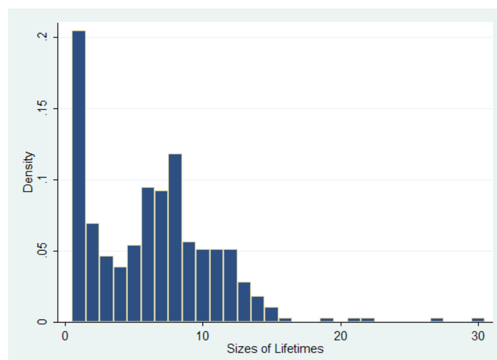
(a) p=0.2



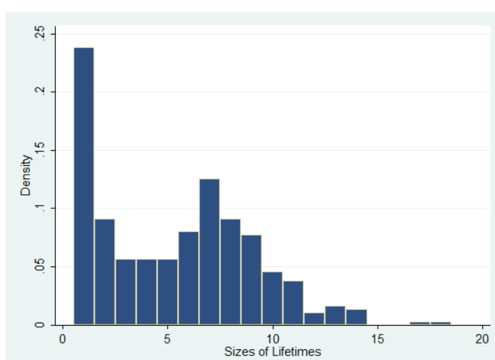
(b) p=0.5



(c) p=0.7

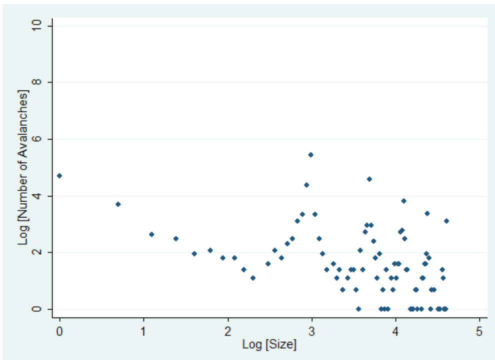


(d) p=0.9

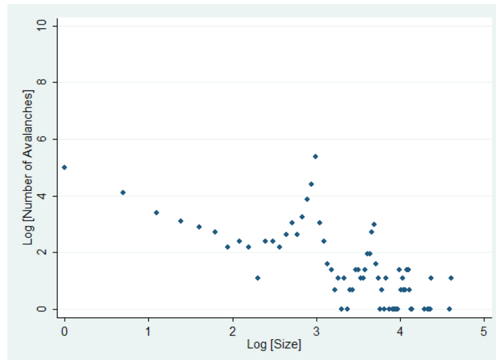


(e) p=1

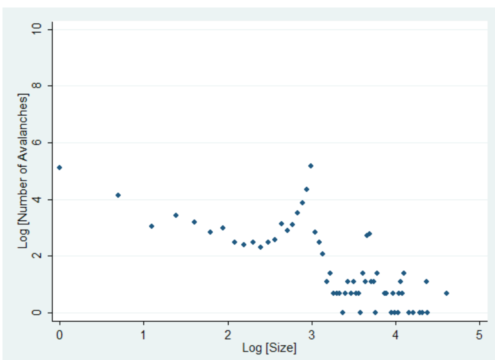
Figure 5.45: The histograms of avalanche lifetimes on a clustered graph with two clusters with 50 vertices for different values of p (probability of sink connection).



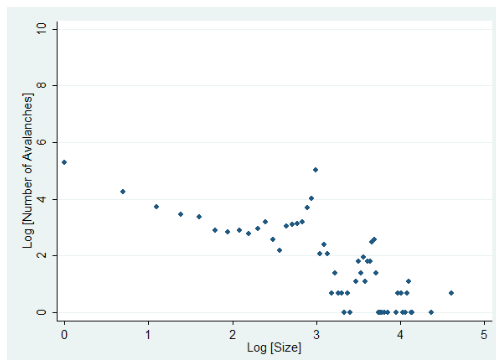
(a) p=0.2



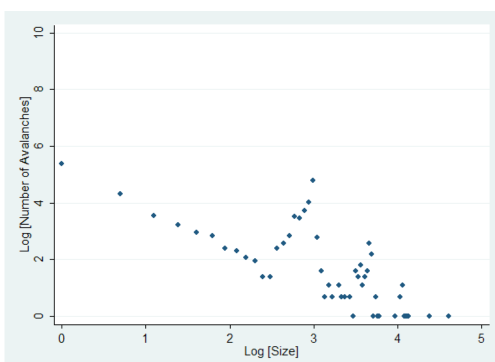
(b) p=0.5



(c) p=0.7

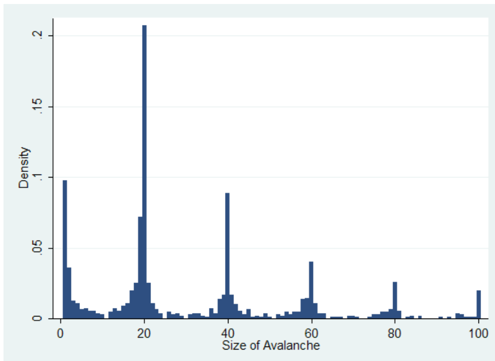


(d) p=0.9

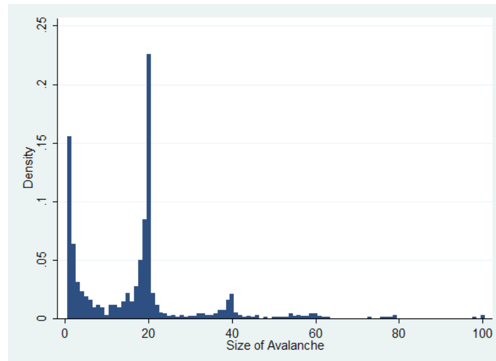


(e) p=1

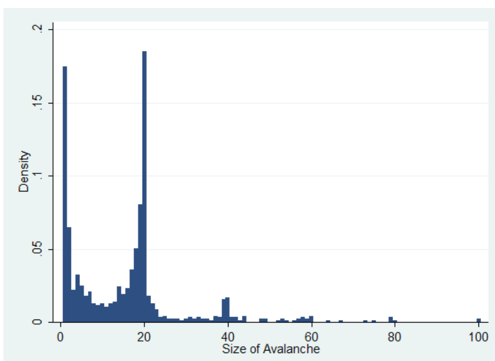
Figure 5.46: The distribution of avalanche sizes on a clustered graph with five clusters with 20 vertices for different values of p (probability of sink connection).



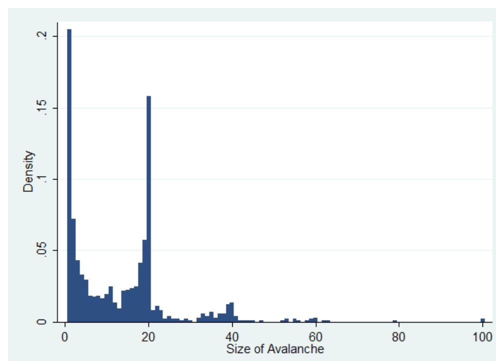
(a) p=0.2



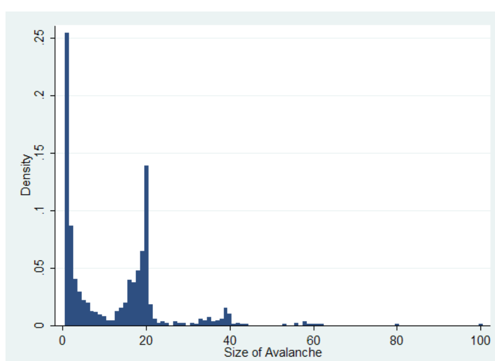
(b) p=0.5



(c) p=0.7

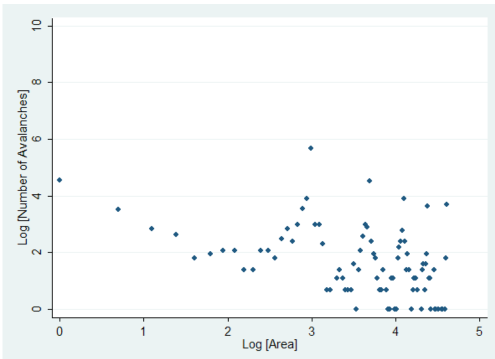


(d) p=0.9

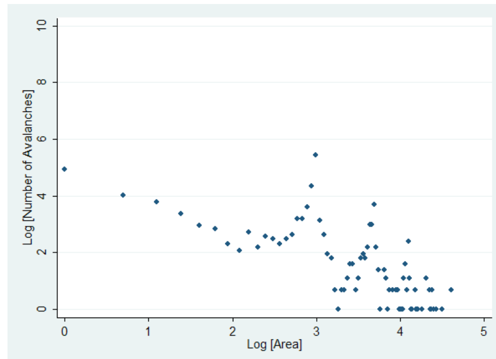


(e) p=1

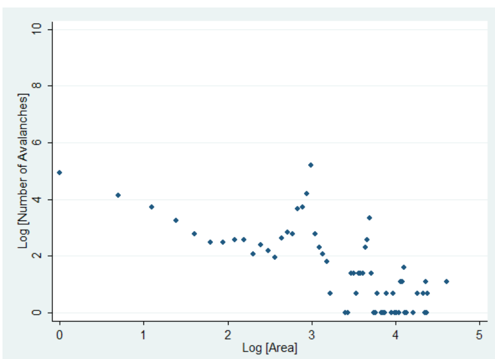
Figure 5.47: The histograms of avalanche sizes on a clustered graph with five clusters with 20 vertices for different values of p (probability of sink connection).



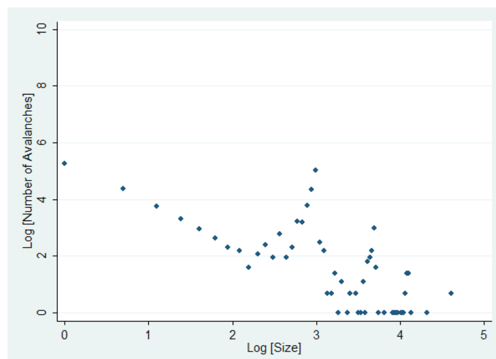
(a) p=0.2



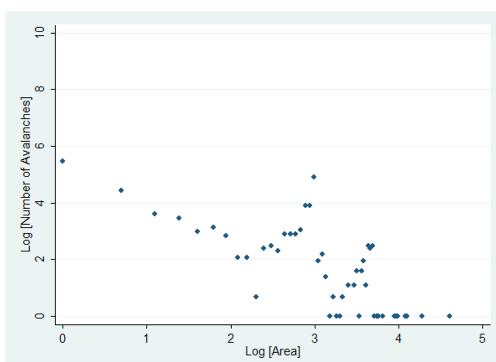
(b) p=0.5



(c) p=0.7

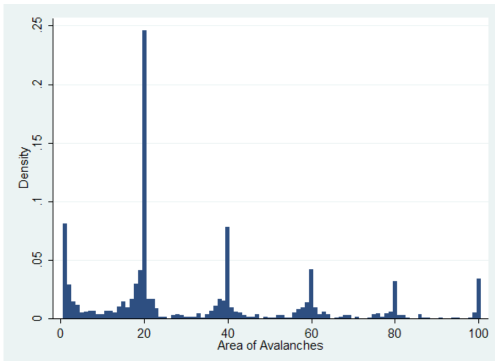


(d) p=0.9

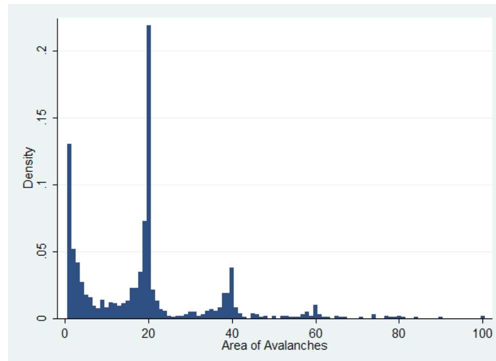


(e) p=1

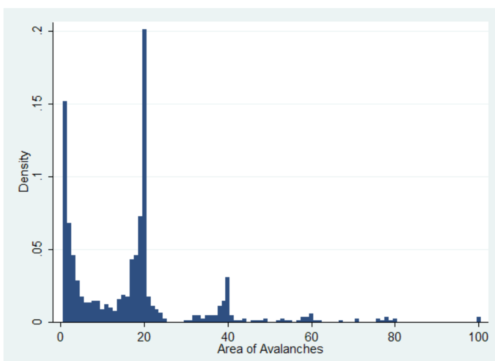
Figure 5.48: The distribution of avalanche areas on a clustered graph with five clusters with 20 vertices for different values of p (probability of sink connection).



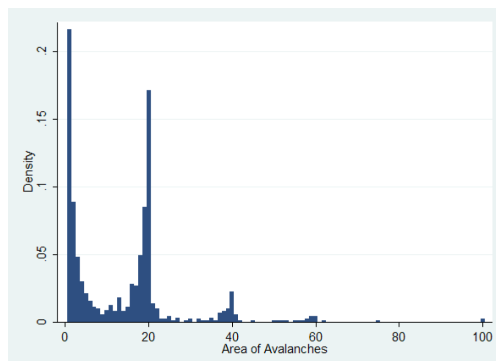
(a) p=0.2



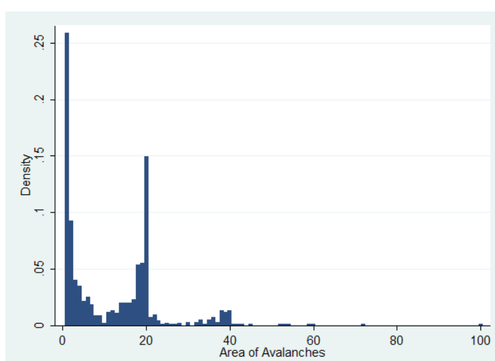
(b) p=0.5



(c) p=0.7

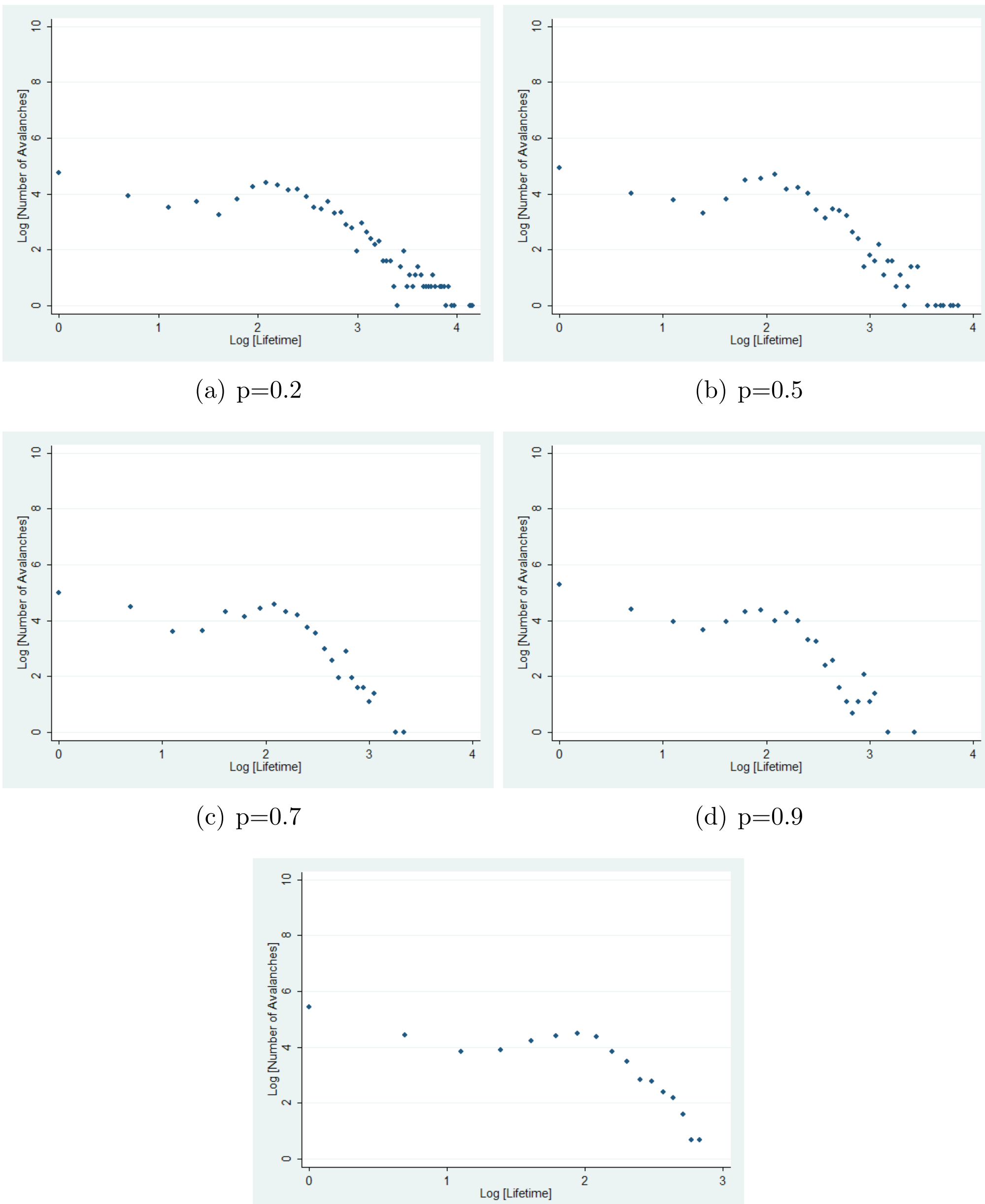


(d) p=0.9



(e) p=1

Figure 5.49: The histograms of avalanche areas on a clustered graph with five clusters with 20 vertices for different values of p (probability of sink connection).



(a) p=0.2

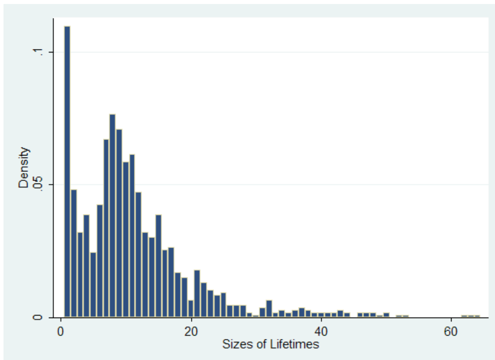
(b) p=0.5

(c) p=0.7

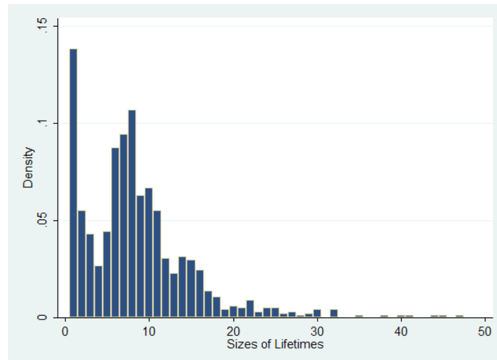
(d) p=0.9

(e) p=1

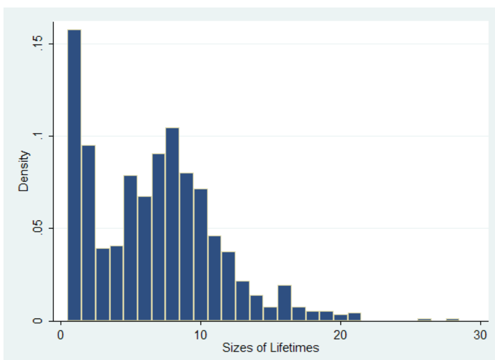
Figure 5.50: The distribution of avalanche lifetimes on a clustered graph with five clusters with 20 vertices for different values of p (probability of sink connection).



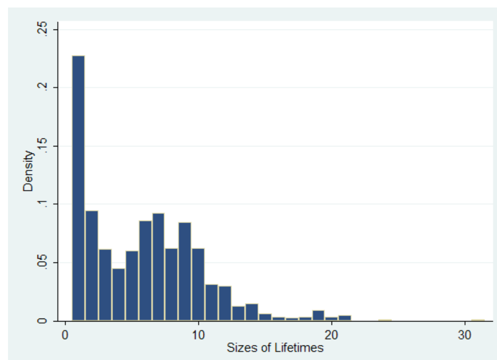
(a) p=0.2



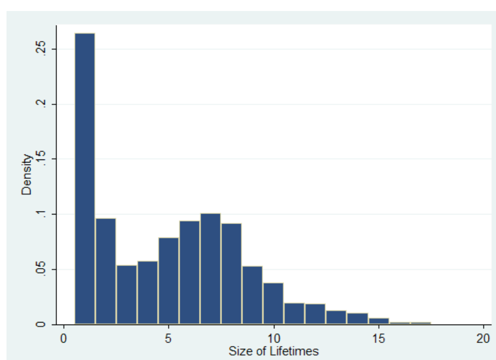
(b) p=0.5



(c) p=0.7

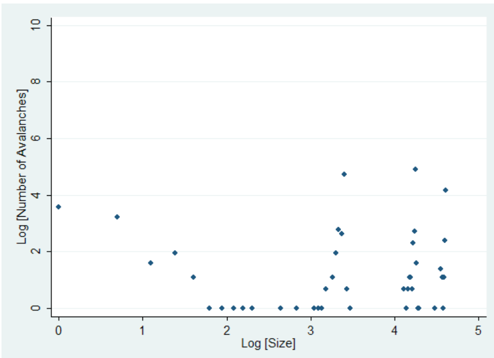


(d) p=0.9

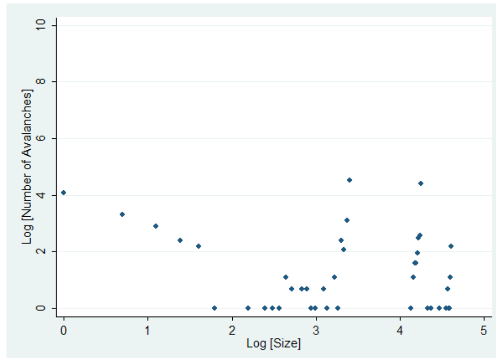


(e) p=1

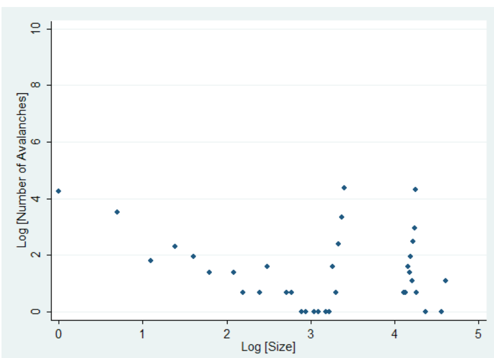
Figure 5.51: The histograms of avalanche lifetimes on a clustered graph with five clusters with 20 vertices for different values of p (probability of sink connection).



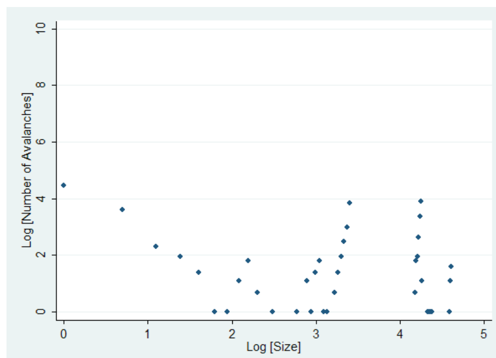
(a) p=0.2



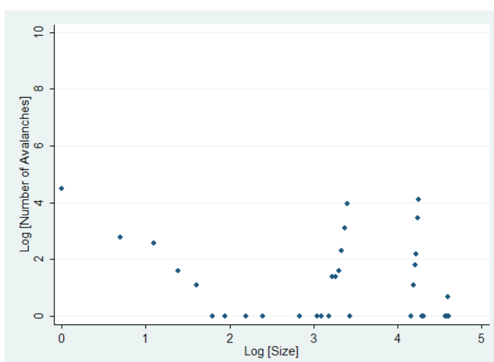
(b) p=0.5



(c) p=0.7

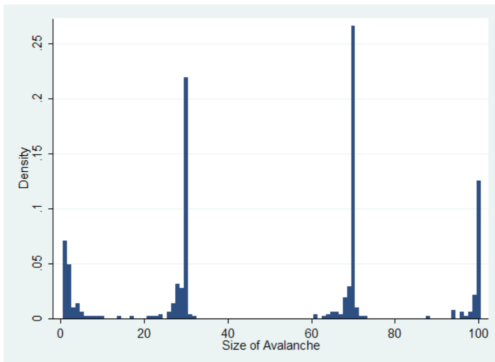


(d) p=0.9

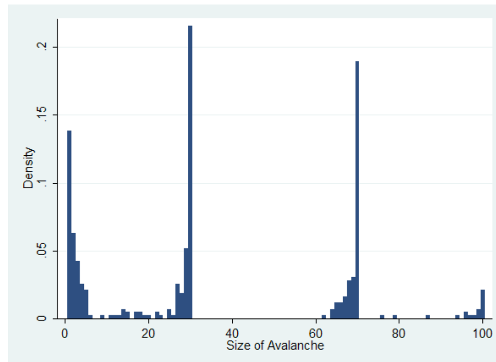


(e) p=1

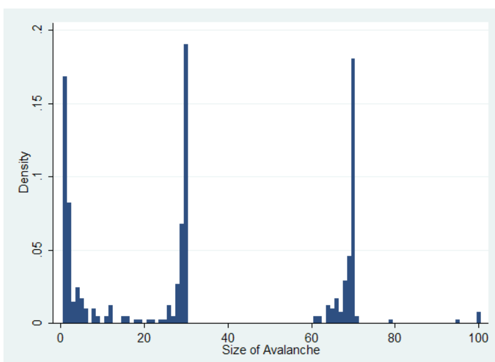
Figure 5.52: The distribution of avalanche sizes on a clustered graph with one cluster of 30 vertices and another of 70 vertices for different values of p (probability of sink connection).



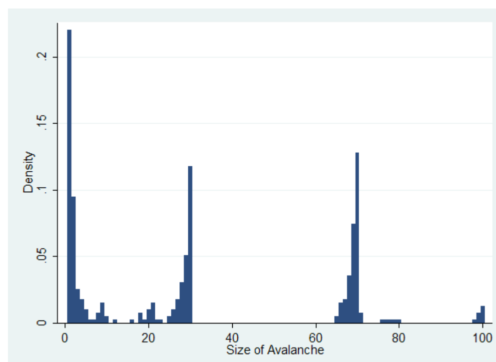
(a) p=0.2



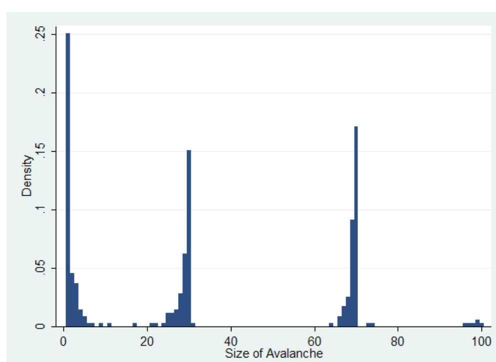
(b) p=0.5



(c) p=0.7

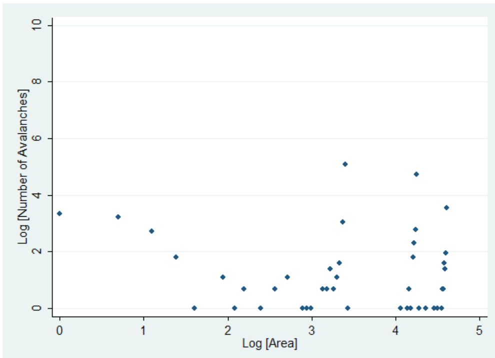


(d) p=0.9

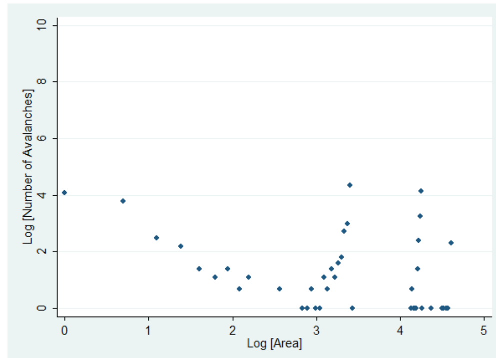


(e) p=1

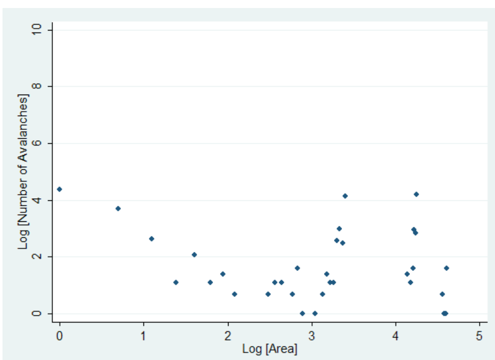
Figure 5.53: The histograms of avalanche sizes on a clustered graph with one cluster of 30 vertices and another of 70 vertices for different values of p (probability of sink connection).



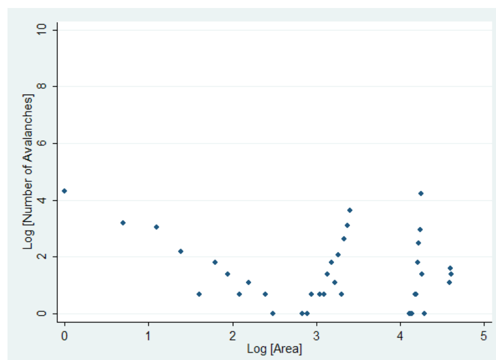
(a) p=0.2



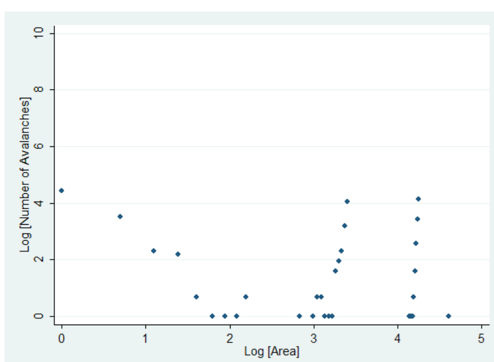
(b) p=0.5



(c) p=0.7

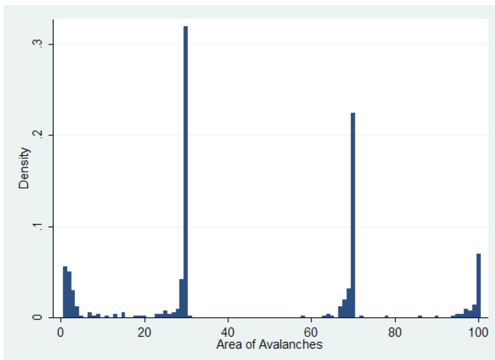


(d) p=0.9

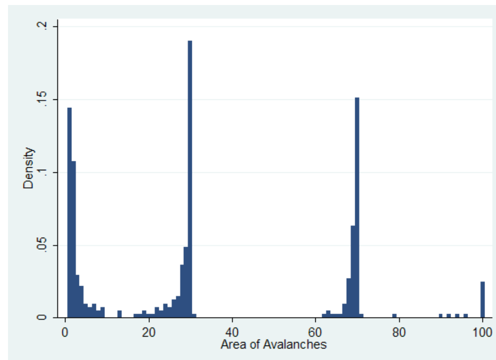


(e) p=1

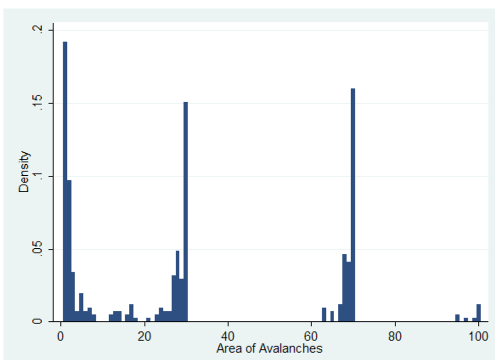
Figure 5.54: The distribution of avalanche areas on a clustered graph with one cluster of 30 vertices and another of 70 vertices for different values of p (probability of sink connection).



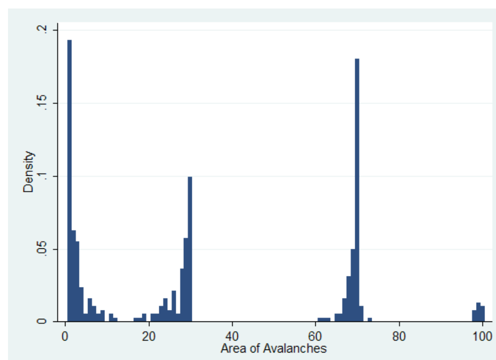
(a) p=0.2



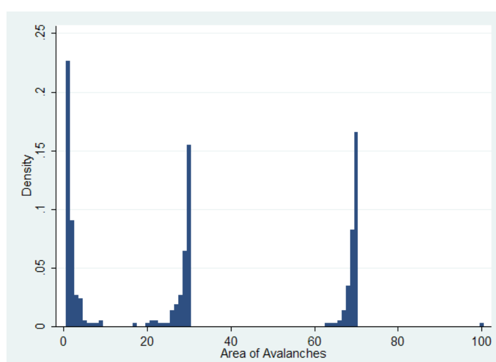
(b) p=0.5



(c) p=0.7

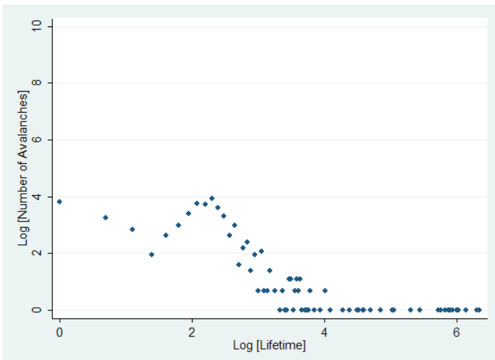


(d) p=0.9

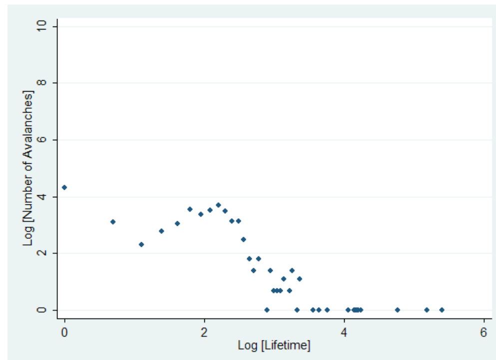


(e) p=1

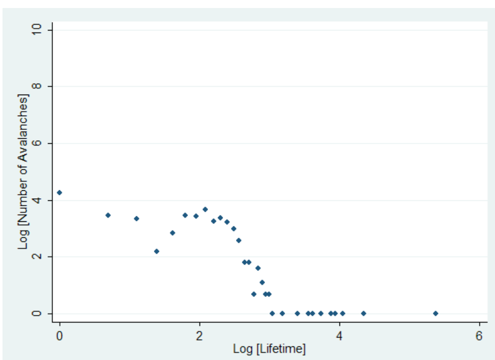
Figure 5.55: The histograms of avalanche areas on a clustered graph with one cluster of 30 vertices and another of 70 vertices for different values of p (probability of sink connection).



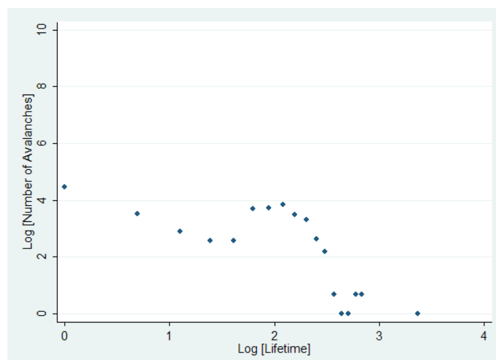
(a) p=0.2



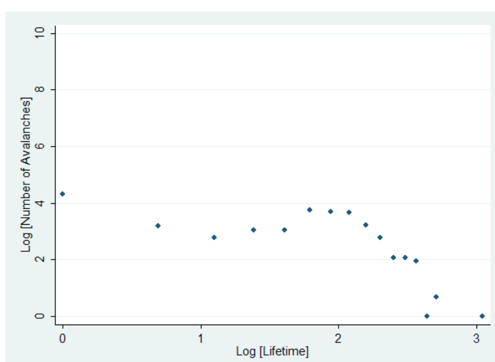
(b) p=0.5



(c) p=0.7

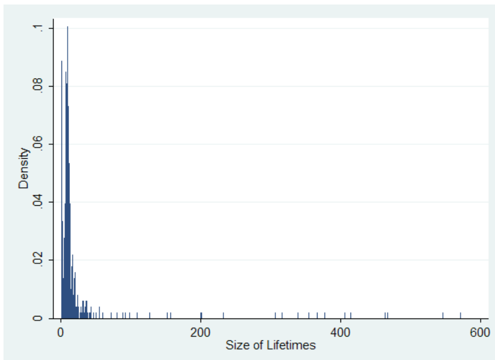


(d) p=0.9

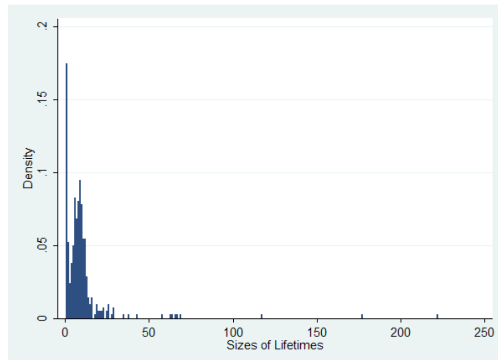


(e) p=1

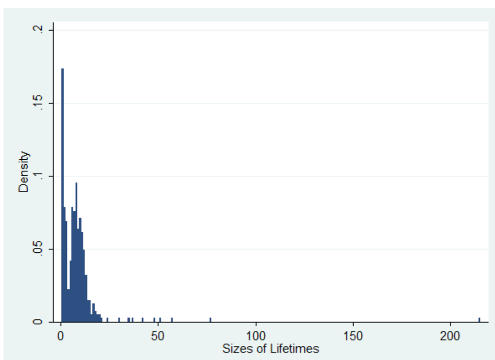
Figure 5.56: The distribution of avalanche lifetimes on a clustered graph with one cluster of 30 vertices and another of 70 vertices for different values of p (probability of sink connection).



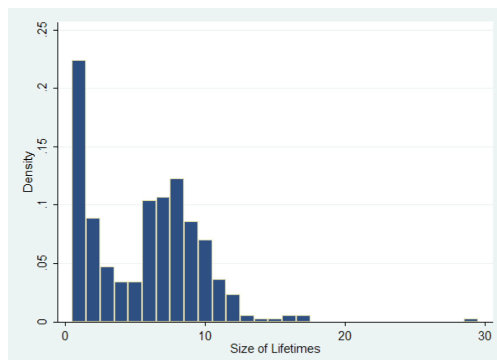
(a) p=0.2



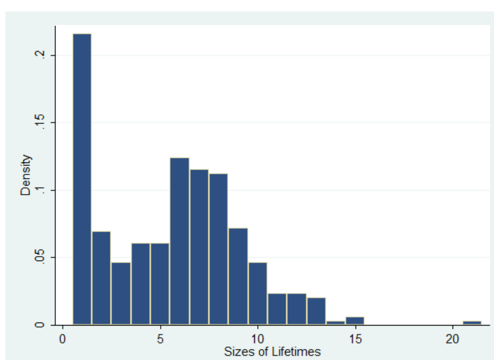
(b) p=0.5



(c) p=0.7



(d) p=0.9



(e) p=1

Figure 5.57: The histograms of avalanche lifetimes on a clustered graph with one cluster of 30 vertices and another of 70 vertices for different values of p (probability of sink connection).

5.7 Random Regular Network

Among the networks considered in this thesis, only the grid and the circular ones were found to exhibit a power-law behavior. The fact that these networks characterized by power-laws consist of vertices with the same degree might suggest that we can only have power laws in the case of similar traders. Therefore, we want to test the emergence of a power-law behavior for the avalanches of a graph which has all the vertices with the same degree, including the sink vertex.[8] We consider a random regular network consisting of an undirected graph with 101 vertices with each vertex of degree d. In order to construct such a network, the product $n * d$ is required to be even since each edge contains two vertices. Moreover, the total number of vertices must be at least $d + 1$. A random regular network of degree 4 on 6 vertices with the sink denoted as vertex 0 is shown in Figure 5.58.

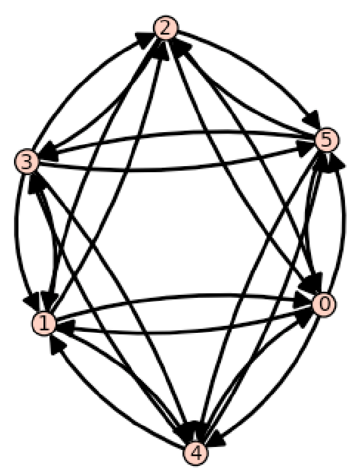
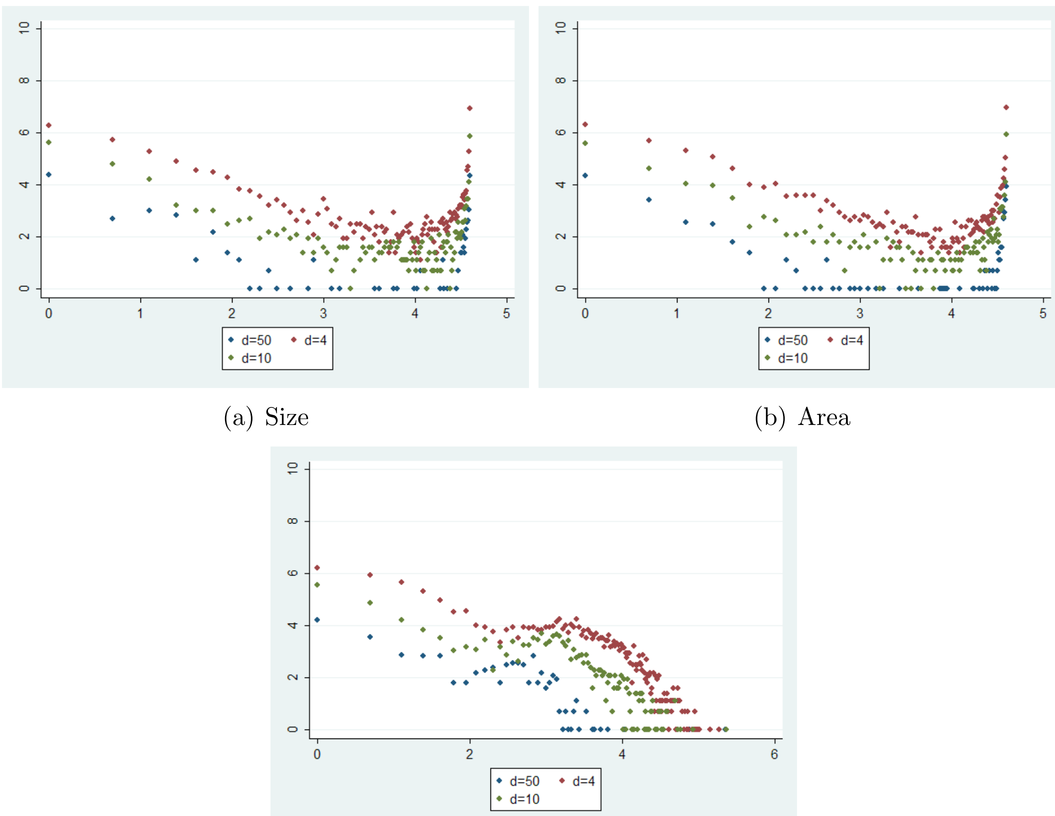


Figure 5.58: The random regular graph of degree 4 on 6 vertices with the sink denoted as vertex 0.

We consider the 4-regular, the 10-regular and the 50-regular graphs and we plot the size, area and lifetime of each avalanche against the number of avalanches of

[8]Recall from Definition 4.3.5 that this is called a regular graph.

that particular strength on a log-log scale. We can see from Figure 5.59 that the areas and avalanches for the regular graph get smaller as the degree of the graph gets bigger. Moreover, there are a lot fewer big avalanches for the 50-regular graph compared to the 4-regular graph. Also, the duration of the avalanches seems to follow the same pattern, since the avalanches get a lot smaller when the degree of the graph gets bigger. The number of avalanches seems to decrease as well for higher degrees. Therefore, it seems to be the case that having a larger number of neighbors leads to a slower transmission of information. We get this result because in this model the number of neighbors a vertex has is the same as the threshold of each cell and thus, we cannot distinguish very well the effect of having more neighbors from the one of having a higher threshold. We can also conclude that none of the measures of the avalanche sizes seems to follow a power-law distribution. This is probably the case since there are too few connections to the sink, so that we actually get a lot more big avalanches than predicted by a power-law behavior.



(a) Size

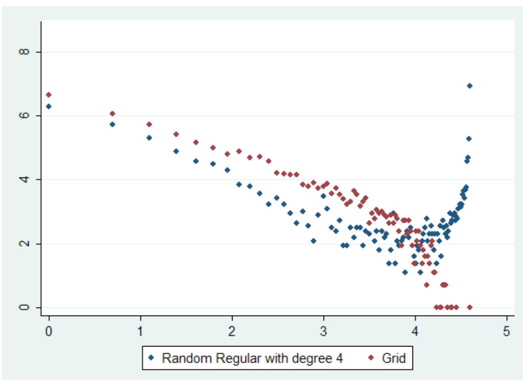
(b) Area

(c) Lifetime

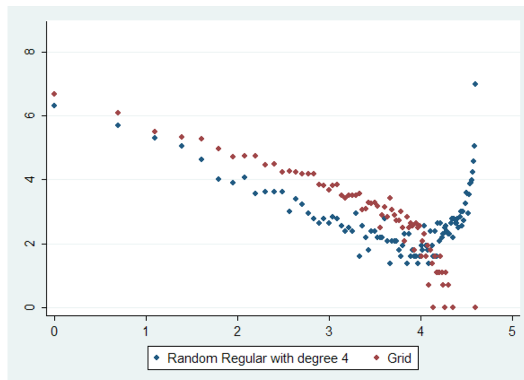
Figure 5.59: The distribution of avalanche strength (size, area, lifetime) on random regular graph of degrees 4, 10 and 50 on a log-log scale.

5.7.1 How is the 4-regular Graph Different from a Grid Graph?

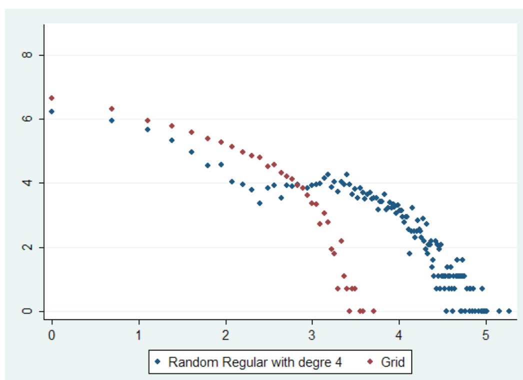
Since the 4-regular graph is somewhat similar to the grid graph, we compare the measures of the avalanche strength in these two graphs. Figure 5.60 shows the plots of the distribution of avalanche strengths on a log-log scale. We can see that the sizes and areas of small avalanches do not differ a lot between the two graphs. Nevertheless, we have a lot more big avalanches in the 4-regular network than in the grid network. Because we have a lot more vertices connected to the sink in the grid graph than in the 4-regular graph (36 compared to 4), we also get smaller sizes and areas of avalanches. Moreover, the duration of the avalanches in the regular graph is found to be longer than for the other graph considered.



(a) Size



(b) Area



(c) Lifetime

Figure 5.60: The distribution of avalanche strength (size, area, lifetime) on 4-random regular graph compares to the one for a grid graph on a log-log scale.

Conclusion

Discussion of the Model

This model makes some assumptions that are hard to observe in reality:

1. In the model, passing along information makes a trader less willing to sell. This assumption is not very intuitive and could only be explained by the high transaction costs that selling incurs.

2. The model does not track identical pieces of information. Information is usually a one-time deal, and thus, it should not matter how many times the information is transmitted, but rather how many people receive the piece of information. Thus, the areas of avalanches could matter more than the sizes.

3. The model does not account for the number of people knowing the piece of information at a given time (how many vertices previously fired). Thus, we need an index to measure the state of the economy at a given time, so that the traders could respond to it. This index should account for the fact that a signal received from a neighbor becomes less important as more people know that piece of information.

4. Real life information is not homogeneous as the grains of sand. This fact is somewhat taken into account by the weighted edges in the random network

considered, which could either measure the influence one trader has on another, or the importance of the piece of information shared.

5. The fact that each vertex's threshold is equal to its out-degree is not a very good assumption, since there might not be a direct connection between the time it takes a trader to share his information and the number of neighbors it has.

6. A vertex receives various grains of sand, but when it fires it only sends one to each neighbor. The question to be asked is what piece of information each neighbor receives and why each one of them receives only one piece. Assuming that all pieces of information are equally relevant is a solution, but it is hard to observe in practice.

7. In the model, transactions costs stay the same throughout the avalanches, which is an oversimplification of reality.

Main Results and Implications

The purpose of this thesis is to explain deviations from the standard finance theory and to assess the possibility of a power-law behavior in the stock market. Before testing the existence of power-laws, this thesis considered the effect of herding behavior and information transmission on the volatility of stock market returns. Since herding behavior and implicitly information transmission appeared to be characterised by a power-law behavior, we consider a model of information transmission and we test whether or not the dynamics of information predicted by this model exhibit power-laws. The model considered was similar to the one in Lee (1998) with the only difference that the experiments were performed using the sandpile model.

The main results of the experiments done in Chapter 5 imply the fact that the type of network of traders is very important for the flow of information. The results shown in Chapter 5 suggest that:

1. The $n \times n$ and the circular networks of traders are the only one from the networks studied that exhibit power law behavior for the strength of avalanches.

2. The dynamics of the circular network do not depend too much on whether the graph is directed or not and they mainly depend on the degree of connectivity of each vertex to the sink.

3. The duration of avalanches in the random graph network are highly dependent on the degree of connectivity of each vertex to the sink and also the probability that two edges are connected.

4. No significant results were obtained for the network with a special node because the activity of the special node in relation to the other nodes was not explicitly isolated.

5. The network composed of various clusters showed more activity in the clusters since they were connected to each other through only one random edge so that the signal rarely traveled form one cluster to the other.

6. The avalanches in the random regular network get smaller and fewer as the degree gets bigger. This is the case because once the degree gets bigger, each vertex has also a bigger threshold.

7. Overall, the connectivity to the sink was found to be very influential for the duration of avalanches, with higher connectivity resulting in smaller avalanches.

The implications of these results for the real-world are important. First, if the network of traders were an $n \times n$ grid with undirected edges or a circular network (directed or undirected) with degree of connectivity to the sink of 1, then the distribution of the number of traders sharing information, the number of times information is shared and the duration of an informational cascade would follow a power-law behavior. Moroever, if the network of traders were a circular graph and each trader would only be connected to two other traders, then information stickiness corresponding to the unwillingness of each person to share his private information would matter a lot. Furthermore, if the network of traders would be represented by a random graph with random connections among traders, then the duration of each informational cascade will depend on both the degree of connectivity with other traders and also the degree of information stickiness.

Since we are inclined to think that the network of traders consists of clusters, the results of the last experiment are really important because they suggest that activity (i.e. transmission of information) within one cluster rarely affects the activities in the clusters when there is only one link between the clusters. We could also infer that adding more links between clusters would increase the probability that the information is transmitted from one cluster to another. Moreover, the random regular networks suggest that the transmission of information gets faster in the case of smaller transaction costs for each trader and less neighbors. Therefore, the transaction costs seem to be more important for sharing information that the number of neighbors each trader.

Directions for Future Research

There are many ways this information transmission model could be improved:

1. First of all, the fact that the power-law distribution is a straight line on a log-log plot does not imply that the reverse is true. Actually, there might be other distributions that give a straight line on a log-log plot.

2. There are many different types of networks that could be considered. Dynamic networks (the ones in which the connections change over time) could also be

considered since they model the real-world in a better way. One important dynamic model is the preferential attachment in which new nodes are added and they form connections to the most “popular” vertices (i.e. the ones with a higher degree of connectivity to other vertices).

3. Another possibility is to extend the model by adding another type of sand so that the traders could also have the option of buying a stock.

4. The idea behind the transmission of information in this model is to have a first mover who sets the behavior of others. The fact that a network can have more than one first mover (more than one grain of sand being dropped at random at the same time or at different times) should be investigated further in an experiment.

5. A general concern about the model considered is that graphical analysis could be deceiving. Therefore, a deeper investigation is necessary in order to check the results.

6. Last but not least, the problems of the applicability of the model in reality discussed in the previous section should be solved.

Appendix A

Important Definitions and Theorems

Definition A.0.1. The cumulative distribution function (c.d.f.) of the random variable X (also called the distribution function F) is the probability that the random variable X takes on a value that is less than or equal to b. It is defined for all real numbers x, with $-\infty < x < \infty$ as:

$$F(x) = P[X \leq x]$$

Remark A.0.2. The cumulative distribution function F exhibits the following properties:

1. If $a < b$, then $F(a) \leq F(b)$.
2. $\lim_{b \to \infty} F(b) = 1$.
3. $\lim_{b \to -\infty} F(b) = 0$.
4. $P[a < X \leq b] = F(b) - F(a)$ for all $a < b$.

Definition A.0.3. The complementary cumulative distribution function (c.c.d.f.) is defined as:

$P[X > x] = 1 - F(x)$, where F is the cumulative distribution function.

Definition A.0.4. A random variable is said to be discrete if it can take on at most a countable number of possible values. Any discrete random variable X is characterized by a probability mass function:

$$P(a) = P[X = a].$$

Definition A.0.5. A random variable X is said to be a continuous random variable if there exists a non-negative function f , defined for all real $x \in (-\infty, \infty)$, such that for any set B of real numbers:

$$P[X \in B] = \int_B f(x)\, dx$$

The function f is called the probability density function of the random variable X.

Remark A.0.6. The function f defined above must satisfy

$$P[X \in (-\infty, \infty)] = \int_{-\infty}^{\infty} f(x)\, dx = 1$$

Definition A.0.7. The expected value (also called the mean of X) of a discrete random variable X with probability mass function $p(x)$ is given by the following formula:

$$E[X] = \sum_x xP[X = x].$$

The expected value of a continuous random variable X with probability density function $f(x)$ is given by the following formula:

$$E[X] = \int_{-\infty}^{\infty} x f(x)\, dx.$$

Definition A.0.8. The variance of a random variable X is given by the formula:

$$Var(X) = \sigma^2 = E[(X - \mu])^2].$$

The square root of the variance is called the standard deviation of X.

Definition A.0.9. The covariance between two random variables X and Y is defined by the following formula:

$$Cov(X, Y) = E[(X - E[X]](Y - E[Y])].$$

Definition A.0.10. The correlation of two random variables X and Y is defined as:

$$\rho(X, Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}, \text{ with } Var(X), Var(Y) > 0.$$

Definition A.0.11. A random variable X is called a normal random variable or simply normally distributed with mean μ and variance σ^2 if its density is given by the following formula:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} \quad -\infty < x < \infty$$

Definition A.0.12. Two random variables X and Y are said to be independent if for any two sets of real numbers A and B, we have:

$$P[X \in A, Y \in B] = P[X \in A]P[Y \in B].$$

Definition A.0.13. Two random variables X and Y are said to be independent and identically distributed (i.i.d.) if they have the same distribution function and are independent.

Definition A.0.14. If a random variable X is normally distributed with mean μ and variance σ^2, then $Z = (X - \mu)/\sigma$ is normally distributed with mean 0 and variance 1 and it is said to have the standard normal distribution.

Theorem A.0.15. ***The Central Limit Theorem***

Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables each having mean μ and variance sigma^2. Then the distribution of

$$\tfrac{X_1+\cdots+X_n-n\mu}{\sigma\sqrt{n}}$$

converges to the standard normal distribution as n approaches ∞.

Definition A.0.16. The skewness of the probability distribution of a random variable X is given by the following formula:

$$\tfrac{E[(X-E[X])^3]}{\sigma^3}.$$

Definition A.0.17. The kurtosis of the probability distribution of a random variable X is given by the following formula:

$$\tfrac{E[(X-E[X])^4}{\sigma^4}.$$

Definition A.0.18. Let $X_1, \ldots, X_n$ be a sequence of i.i.d. random variables such that X_i takes the values -1 and $+1$ only, with $P[X_i = +1] = P[X_i = -1] = 1/2$. Then the sequence

$$S_n = \sum_{i=1}^{n} X_i, \text{ with } S_0 = 0 \text{ is called a random walk.}$$

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