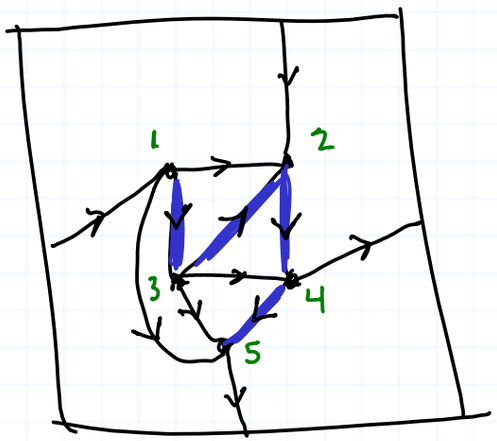
Math 412

$G = K_5$ on a torus

$K(G) = \left(\frac{\mathbb{Z}}{5\mathbb{Z}}\right)^3$



Basis for cutspace of G

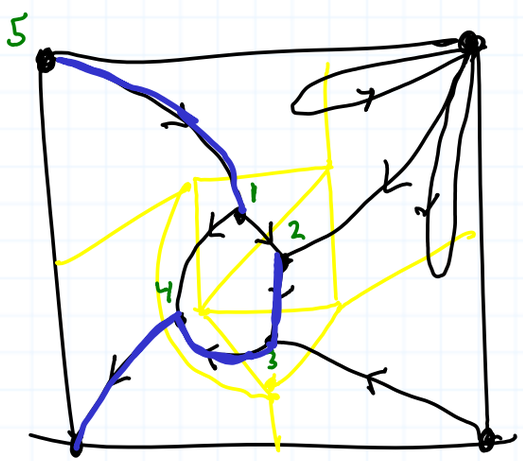
	$\overline{12}$	$\overline{13}$	$\overline{14}$	$\overline{15}$	$\overline{23}$	$\overline{24}$	$\overline{25}$	$\overline{34}$	$\overline{35}$	$\overline{45}$
c^*_{13}	1	1	-1	1	0	0	0	0	0	0
c^*_{23}	1	0	-1	1	1	0	0	1	1	0
c^*_{24}	0	0	-1	1	0	1	-1	1	1	0
c^*_{45}	0	0	0	1	0	0	-1	0	1	1

Cutspace for G^*

c^*_{51}	-1	1	0	0	1	0	0	0	0	0
c^*_{23}	0	0	0	0	-1	-1	0	1	0	0
c^*_{34}	0	0	0	0	-1	-1	0	0	1	-1
c^*_{45}	0	1	0	-1	1	1	0	0	0	1

G^*

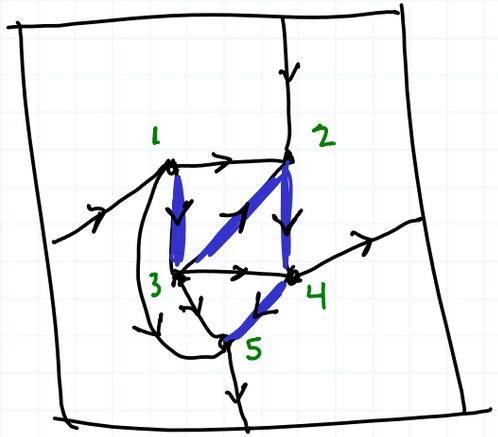
$\left(\frac{\mathbb{Z}}{3\mathbb{Z}}\right)^2 \times \frac{\mathbb{Z}}{5\mathbb{Z}} = K(G^*)$



$$\frac{\mathbb{Z}E}{c^*_{G^*} + c^*_G} \cong \frac{\mathbb{Z}}{5\mathbb{Z}} \times \mathbb{Z}^2$$

$$\frac{\mathbb{Z}E}{c_G + c^*_G} \cong \left(\frac{\mathbb{Z}}{5\mathbb{Z}}\right)^3$$

$$\frac{\mathbb{Z}E}{c^*_{G^*} + c_{G^*}} \cong \left(\frac{\mathbb{Z}}{3\mathbb{Z}}\right)^2 \times \frac{\mathbb{Z}}{5\mathbb{Z}}$$



$\mathcal{C}_G$

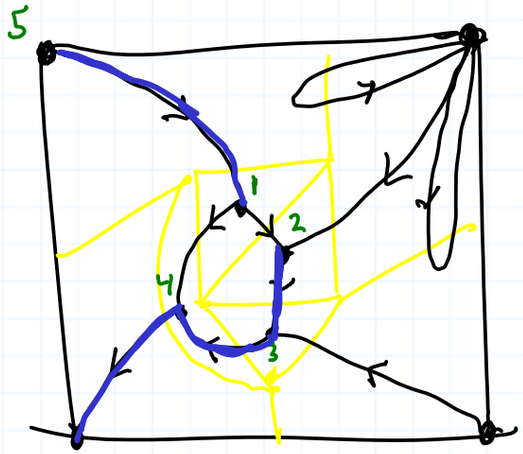
Cycle space for G

	$\overline{12}$	$\overline{13}$	$\overline{14}$	$\overline{15}$	$\overline{23}$	$\overline{24}$	$\overline{25}$	$\overline{34}$	$\overline{35}$	$\overline{45}$
c_{12}	1	-1	0	0	-1	0	0	0	0	0
c_{14}	0	1	1	0	1	1	0	0	0	0
c_{15}	0	1	0	-1	1	1	0	0	0	1
c_{25}	0	0	0	0	0	1	1	0	0	1
c_{34}	0	0	0	0	1	1	0	-1	0	0
c_{35}	0	0	0	0	1	1	0	0	-1	1

$$0 \to \mathcal{C}^*_{G^*} \to \mathcal{C}_G \to \mathbb{Z}^2 \to 0$$

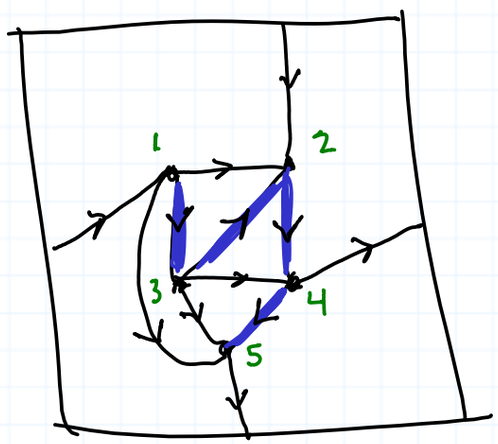
* c^*_{51}	-1	1	0	0	1	0	0	0	0	0	=	$-c_{12}$
c^*_{23}	0	0	0	0	-1	-1	0	1	0	0	=	$-c_{34}$
c^*_{34}	0	0	0	0	-1	-1	0	0	1	-1	=	$-c_{35}$
c^*_{45}	0	1	0	-1	1	1	0	0	0	1	=	c_{15}

$\dfrac{\mathcal{C}_G}{\mathcal{C}^*_{G^*}}$ is generated by c_{14} and c_{25}



Cycles for G^*

	$\overline{12}$	$\overline{13}$	$\overline{14}$	$\overline{15}$	$\overline{23}$	$\overline{24}$	$\overline{25}$	$\overline{34}$	$\overline{35}$	$\overline{45}$
c_{12}	1	0	0	1	1	0	0	1	1	0
c_{14}	1	1	0	1	0	0	0	0	0	0
c_{25}	0	0	0	1	0	1	0	1	1	0
c_{35}	0	0	0	1	0	0	0	0	1	1
c_{55}	0	0	0	0	0	0	1	0	0	0
c'_{55}	0	0	1	0	0	0	0	0	0	0

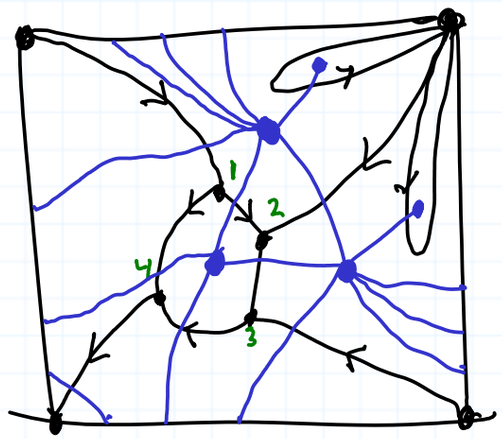


$$0 \to C^*_G \to C_{G^*} \longrightarrow \mathbb{Z}^2 \to 0$$

c^*_{13}	1	1	-1	1	0	0	0	0	0	0	$= c_{14} - c'_{55}$
c^*_{23}	1	0	-1	1	1	0	0	1	1	0	$= c_{12} - c'_{55}$
c^*_{24}	0	0	-1	1	0	1	-1	1	1	0	$= c_{25} - c_{55} - c'_{55}$
c^*_{45}	0	0	0	1	0	0	-1	0	1	1	$= c_{35} - c_{55}$

C_{G^*}/C^*_G generated by c_{55} and c'_{55}

$G^{**} \neq G$



So we would have

$$\mathbb{Z}E \twoheadrightarrow K(G^{*})$$

$$C^{*}_{G^{*}} + C^{*}_{G^{**}} \twoheadrightarrow K(G^{**})$$

What about G^{***}, etc. ?