Math 374 HW, due Monday, March 9

1. Show that if $\phi\colon G \to G'$ is a non-constant harmonic mapping, then ϕ is surjective on vertices and edges. (It follows that the induced mapping on Picard and Jacobian groups is surjective.)

2. If $\phi\colon G \to G'$ is a non-constant harmonic mapping, show:
   (a) $\deg \phi^*(D') = \deg \phi \deg(D')$ for all $D' \in \mathrm{Div}(G')$;
   (b) $\phi_* \phi^*$ is multiplication by $\deg \phi$.

3. Create a degree 4 harmonic mapping $\phi : G \to G'$ where $G' = C_4$, the cycle graph on 4 vertices.
   (a) Draw a picture of this (as we did in class Monday, week 6).
   (b) Verify the Riemann-Hurwitz theorem $K = \phi^* K' + R$ where R is the ramification divisor (introduced in class on Monday, week 6) by calculating K, K', $\phi^* K'$ and R.
   (c) Verify that $2g-2 = \deg(\phi)(2g'-2)+2\sum_{v\in V}(m_v-1)+\sum_{v\in V}\mathrm{vert}(v)$.
   (d) Use Sage to compute the invariant factors of G.

4. Create a degree 2 harmonic mapping $\phi\colon G \to G'$ where G' is our “favorite” graph (formed by removing an edge from K_4).
   (a) Draw a picture of G.
   (b) Verify the Riemann-Hurwitz theorem $K = \phi^* K' + R$ where R is the ramification divisor by calculating K, K', $\phi^* K'$ and R.
   (c) Verify that $2g-2 = \deg(\phi)\,(2g'-2)+2\sum_{v\in V}(m_v-1)+\sum_{v\in V}\mathrm{vert}(v)$.
   (d) Use Sage to compute the invariant factors of G.

Possible project: Is there any way to predict the invariant factors of a covering G of C_n? What happens to the invariant factors as we add identify vertices above a given vertex of C_n or when we add vertical edges?