Math 372 Homework 2

Note: Exercise numbers refer to the 2013 edition of our text.

Chapter 3, exercise 6. Show that if u and v are two vertices of a connected graph G, then we need not have $H(u,v) = H(v,u)$, where H denotes access (hitting) time. What if G is also assumed to be regular?

Chapter 3, exercise 7. Let u and v be distinct vertices of the complete graph K_n. Show that $H(u,v) = n-1$.

Chapter 4, exercise 1. Draw Hasse diagrams of the 16 nonisomorphic four-element posets. For a more interesting challenge, draw also the 63 five-element posets. Those with lots of time to kill, draw the 318 six-element posets, 2045 seven-element posets, the 16999 eight-element posets, up to 4483130665195087 sixteen-element posets.

Chapter 4, exercise 2a. Let P be a finite poset and $f\colon P \to P$ an order-preserving bijection. I.e., f is a bijection (one-to-one and onto), and if $x \le y$ in P then $f(x) \le f(y)$. Show that f is an automorphism of P, i.e., f^{-1} is order-preserving. (Try to use simple algebraic reasoning, though it's not necessary to do so.)

Chapter 4, exercise 4abc. Let q be a prime power, and let F_q denote the finite field with q elements. Let $V := V_n(q) := F_q^n$, the n-dimensional vector space over F_q of n-tuples of elements of F_q . Let $B_n(q)$ denote the poset of all subspaces of V, ordered by inclusion. It's easy to see that $B_n(q)$ is graded of rank n, the rank of a subspace of V being its dimension.

(a) Draw the Hasse diagram of $B_3(2)$. (It has 16 elements.)

(b) Show that the number of elements of $B_n(q)$ of rank k is given by the *q-binomial coefficient*

$$\binom{\mathbf{n}}{\mathbf{k}} = \frac{(q^n-1)(q^{n-1}-1)\cdots(q^{n-k+1}-1)}{(q^k-1)(q^{k-1}-1)\cdots(q-1)}.$$

(c) Show that $B_n(q)$ is rank-symmetric.