Let the following be an exact sequence of groups:

$$1 \to N \xrightarrow{\phi} G \xrightarrow{\psi} H \to 1.$$

1. Prove that ψ is surjective.
2. Prove that ϕ is injective. So replacing N by the image of N, we will consider N to be a subgroup of G and ϕ to be the inclusion mapping, taking an element to itself.
3. Prove that N is a normal subgroup of G.
4. Prove that $H \approx G/N$.
5. Suppose that G is abelian and that there is a mapping (homomorphism) $j \colon H \to G$ such that $\psi \circ j = \mathrm{id}_H$, the identity mapping on H. In this case, prove that $G \approx N \times H$ by describing an explicit isomorphism (with proof).

   (The mapping j is called a *splitting* of the exact sequence. If there is a splitting, the short exact sequence is called *split exact*. The condition is equivalent to the existence of a splitting on the other end of the sequence: a mapping $\iota : G \to N$ such that $\iota \circ \phi = \mathrm{id}_N$. For fun, you can try to prove this equivalence.)