Problems from our text:

- 21.3

- 21.4

- 21.5

- 22.8

- 24.4

More problems:

1. For each of subgroup H of S_3 either (i) show H is not normal, or (ii) describe S_3/H (identify it as isomorphic to some familiar group).

2. There are 4 normal subgroups of S_4:

$$\{()\}, \quad H = \{(), (1,2)(3,4), (1,3)(2,4), (1,4)(2,3)\}, \quad A_4, \quad S_4.$$

   Give a multiplication table for S_4/H. Can you identify this group as something you have seen before?

3. The subgroup of S_4 generated by $(2,3,4)$ and $(3,4)$ has six elements.

   (a) Prove that this subgroup is not normal.

   (b) Find the distinct right cosets of this subgroup.

4. Let A be the subgroup of $\mathbb{Z}^4$ generated by the columns of the matrix

$$L = \begin{pmatrix} 0 & -1 & -1 \\ 4 & -1 & -1 \\ -2 & 2 & 0 \\ -2 & 0 & 2 \end{pmatrix}.$$

   (a) Find matrices U, V, and D, such that D is in Smith normal form and $ULV = D$.

   (b) Give an explicit isomorphism of $\mathbb{Z}^4/A$ and a finitely generated abelian group in standard form:

$$\mathbb{Z}/d_1\mathbb{Z} \times \cdots \times \mathbb{Z}/d_r\mathbb{Z} \times \mathbb{Z}^n$$

   with $d_1|d_2|\cdots|d_r$.

5. **Challenge.** Find all subgroups of index two in $\mathbb{R}^*$, the multiplicative group of nonzero real numbers.