HW 5, due Tuesday, March 3 *Math 332*

Problems from our text:

- 20.8 (Please use the Chinese remainder theorem, Theorem 20.6.)
- 20.10
- 20.11 (There are 14 groups of order 36, by the way.)

More problems:

1. Let A and B be groups. Prove that $A \oplus B$ is abelian iff A and B are abelian.
2. In class we counted the number of different necklaces with six beads and three colors modulo D_6 symmetry.
   (a) How many necklaces as above if we must use exactly two of each color?
   (b) How many necklaces with six beads from three colors but only modulo the rotations, C_6?
3. I used gap to find all the small groups of order 60 (since the question cam up in class). To do this, I first loaded a library of groups by entering the command

```
sage -i database_gap-4.4.10
```

in a terminal window while connected to the Internet (this step only needs to be done once). From the terminal, I then started GAP with

```
sage -i gap
```

and then

```
gap> SmallGroupsInformation(60);

  There are 13 groups of order 60.
  They are sorted by their Frattini factors.
     1 has Frattini factor [ 30, 1 ].
     2 has Frattini factor [ 30, 2 ].
     3 has Frattini factor [ 30, 3 ].
     4 has Frattini factor [ 30, 4 ].
     5 - 13 have trivial Frattini subgroup.
```

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  For the selection functions the values of the following attributes
  are precomputed and stored:
     IsAbelian, IsNilpotentGroup, IsSupersolvableGroup, IsSolvableGroup,
     LGLength, FrattinifactorSize and FrattinifactorId.

  This size belongs to layer 2 of the SmallGroups library.
  IdSmallGroup is available for this size.

gap> List(AllSmallGroups(60), i -> IsAbelian(i));
[ false, false, false, true, false, false, false, false, false, false, false,
  false, true ]
```

PROBLEM. Describe 4 of these (you have names for two nonabelian ones already). Prove your examples are non-isomorphic. (Remember that the Chinese remainder theorem is "iff".)

4. (See the notes from last Thursday's class or the proof of Theorem 20.6.) Give the inverse of the isomorphism

$$\begin{aligned} \phi\colon \mathbb{Z}_{420} &\to \mathbb{Z}_4 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \times \mathbb{Z}_7 \\ a &\mapsto (a \bmod 4, a \bmod 3, a \bmod 5, a \bmod 7) \end{aligned}$$

5. If G is a finite abelian group with an odd number of elements, prove that the product of all elements of G is the identity.