HW 4, due Tuesday, February 24

Problems from our text:

- 14.10
- 14.21
- 14.23
- 15.11
- 16.14

More problems:

- 1. Use (the proof of) Cayley's theorem, Theorem 15.6, to realize  $U_8$  as a subgroup of a symmetric group.
- 2. Prove Proposition 16.13. Note there is a typo in part (1): g should be a.
- 3. Fix and integer n > 1, and let k be an integer with 1 < k < n. When is the function  $x \mapsto x^k$  a permutation of  $S_n$ ? Use Sage to look for patterns. (I don't know the answer.) Find the smallest n for which there exists a nonprime k giving an permutation. (By "permutation of  $S_n$ " I mean a bijection of  $S_n$  with itself.)

Here is some useful Sage code:

```
sage: def f(x,n):
....: return x^n
....:
```

(The dots will be provided by Sage automatically. Note the semicolon.)

```
sage: x = [i^2 for i in SymmetricGroup(4)]
sage: len(x)
24
sage: x = set(x)
sage: len(x)  # note: x originally had repeats
12
```