HW 3, due Tuesday, February 17 *Math 332*

Problems from our text:

- 12.14
- 12.23
- 13.5
- 13.7

More problems:

1. Let H be a subgroup of a group G, and suppose $g \in G$ has order n. If $g^m \in H$ and $\gcd(m, n) = 1$, show that $g \in H$.
2. Draw the subgroup lattice for $\mathbb{Z}_{100}$.
3. What is the order of 35 in $\mathbb{Z}_{100}$?
4. Show that A_8, the alternating group of even permutations in S_8 has an element of order 15.
5. What is the maximum order of any element in A_{10}?
6. Use Sage to find the order of the permutation group generated by $(1, k)$ and $(1, 2, \ldots, n)$ for various n and $1 \leq k \leq n$. Conjecture a necessary and sufficient condition on k and n so that they generate S_n. Sample code:

   ```
   sage: PermutationGroup([[(1,3)], [(1,2,3,4)]]).order()
   8
   sage: factorial(4)
   24
   ```

   (I find the huge number of brackets involved here to be an easy source of syntax errors.)
7. Challenge problem: Suppose G is a group with exactly one nontrivial proper subgroup. Prove that G is cyclic of order p^2 for some prime p. (*Nontrivial* means not the identity subgroup, and *proper* means not the whole group.)