HW 2, due Tuesday, February 10 *Math 332*

Problems from our text:

- 7.7
- 9.4
- 9.6
- 9.8

More problems:

1. Find an example of a group, G, elements $a, b \in G$ and an integer $n \geq 2$ such that $(ab)^n = a^n b^n$ but $ab \neq ba$.
2. Let G be a group and suppose $a^2 = 1$ for all $a \in G$. Prove that G is abelian.
3. Compute the centers of S_3 and D_4.
4. Compute and draw the subgroup lattice for S_3.
5. Find all elements of $\mathrm{GL}(2, 2)$, the 2×2 matrices over $\mathbb{Z}_2$ with nonzero determinant.
6. Sage
   (a) Use Sage to compute the order of $\mathrm{SL}(2, p)$ and the order of $\mathrm{GL}(2, p)$ for lots of primes. [See the Sage Reference Manual, section 24.10 for how to create these groups. For instance, `G = GL(2,5)` creates the general linear group of 2×2 matrices over the field $\mathbb{Z}_5$, then `G.order()` gives the number of elements (this can be done in one step: `GL(2,5).order()`). Note that `primes(2,30)` will list lots of primes.] Report some of your results.
   (b) Make a conjecture for the number of elements in $\mathrm{SL}(2, p)$ for p prime. [The online encyclopedia of integer sequences—Google it—might be useful.]
   (c) Make a conjecture for the number of elements in $\mathrm{GL}(2, p)$ for p prime. [Hint: what is the index of $\mathrm{SL}(2, p)$ in $\mathrm{GL}(2, p)$, i.e., $|\mathrm{GL}(2, p)|/|\mathrm{SL}(2, p)|$?]