Math 332 HW 1

From the text:

1.2. Show the n^{th} roots of 1 in $\mathbb{C}$ form a group.

Pf/ $1^n = 1$, so the group has the identity element, 1.
I'll assume we know multiplication of complex numbers is associative. If $z^n = 1$ and $w^n = 1$, then $(zw)^n$.
If $z^n = 1$, then $(z^{-1})^n = \frac{1}{z^n} = 1$. Thus, each n^{th} root of 1 has an inverse. $\square$

2.5 If G is a group and $a \in G$, show $(a^{-1})^{-1} = a$.

Pf/ $a \cdot (a^{-1}) = (a^{-1}) a = e \implies a = (a^{-1})^{-1}$. $\square$

2.6. For G a group and $a,b \in G$, show $(ab)^{-1} = b^{-1}a^{-1}$.

Pf/ $(b^{-1}a^{-1})(ab) = b^{-1}(a^{-1}a)b = b^{-1}eb = b^{-1}b = e$

and $(ab)(b^{-1}a^{-1}) = e$. $\square$

2.8. Let G be a group and $a \in G$. Show that the conjugation mapping $C_a : G \to G$ defined by $C_a(g) = aga^{-1}$ [I have been using $a^{-1}ga$ in class] is bijective w/ inverse $C_{a^{-1}}$.

Pf/ $C_a C_{a^{-1}}(g) = a(a^{-1}g(a^{-1})^{-1})a^{-1} = aa^{-1}gaa^{-1}$ (since $(a^{-1})^{-1} = a$)

$= g$

and similarly,

$C_{a^{-1}}C_a(g) = a^{-1}(aga^{-1})(a^{-1})^{-1} = g$. $\square$

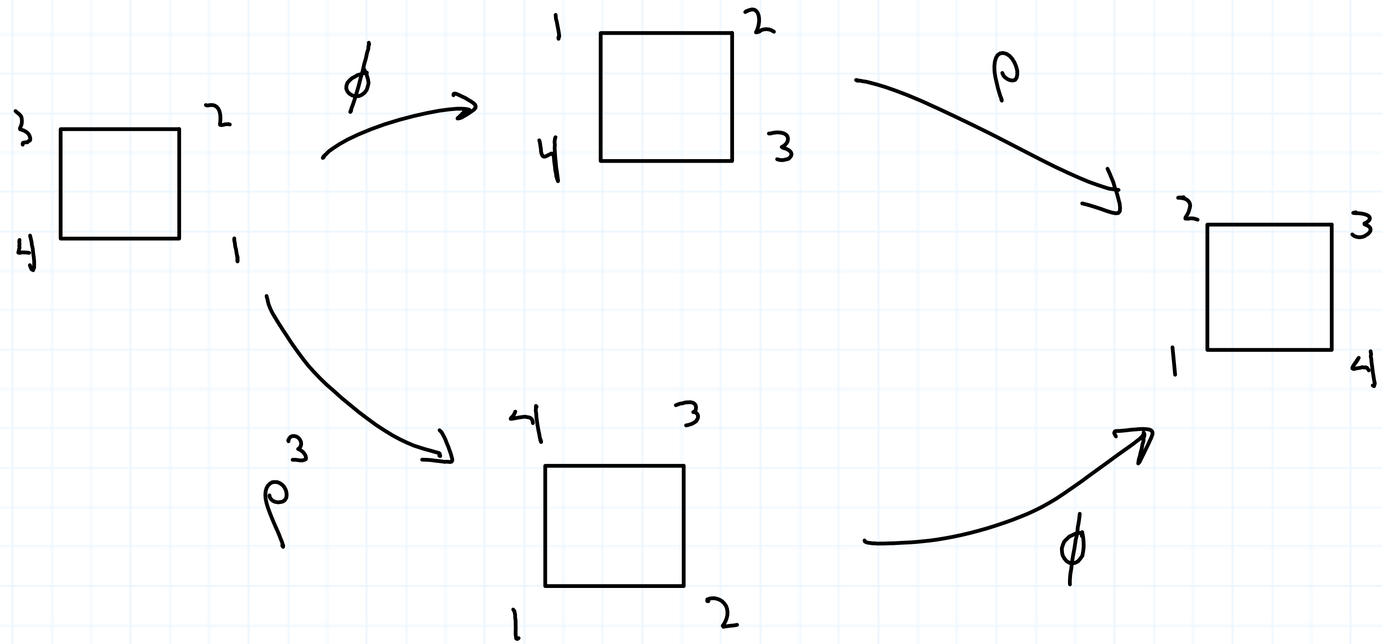
This is enough, although some people probably did this the long way.

1. Suppose G is a finite group with identity e, and $a \in G$. Say G has n elements, and consider

$$a, a^2, \ldots, a^{n+1}.$$

The pigeon-hole principle says $\exists\ n+1 \geq j > i \geq 1$ such that $a^j = a^i$. Then $a^{j-i} = e$. $\square$

2. (a)



2 (b) etc.

(I guess some people made systematic mistakes. Please grade this a 0 and ask for the exercise to be re-submitted for full credit.)

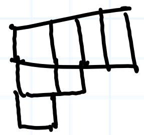
(c)

$$\begin{tikzcd}
() \arrow[r, "\rho"] \arrow[d, leftrightarrow, "\phi"'] & (1234) \arrow[r, "\rho"] \arrow[d, leftrightarrow, "\phi"'] & (13)(24) \arrow[r, "\rho"] \arrow[d, leftrightarrow, "\phi"'] & (1432) \arrow[lll, bend right, "\rho"'] \arrow[d, leftrightarrow, "\phi"'] \\
(13) \arrow[rrr, bend right, "\rho"'] & (12)(34) \arrow[l, "\rho"'] & (24) \arrow[l, "\rho"'] & (14)(23) \arrow[l, "\rho"']
\end{tikzcd}$$

A cube, actually.

3. For the Sage problem give 4/5 for just reporting some correct results. The general pattern might be a little tricky for them to state. 5

There is one conjugacy class for each partition of n.

For example, $7 = 4+2+1 \rightarrow$ 

which corresponds to the conjugacy class in S_7 represented by the element $(1234)(56)$

The "1" corresponds to (7), which is omitted, usually, in cycle notation.