Problems from our text:

- 1.2
- 2.5
- 2.6
- 2.8

More problems:

1. Let G be a finite group with identity e, and let $a \in G$. Prove that $a^n = e$ for some positive integer n. (You'll probably want to use the pigeon-hole principle. See the Wikipedia if you don't know what this means).

2. On the first day of class we considered D_4, the group of symmetries of a square. The element $\rho = (1, 2, 3, 4)$ represented a rotation and $\phi = (1, 3)$ a flip about the diagonal. The elements of D_4 are
$$1, \rho, \rho^2, \rho^3, \phi, \rho\phi, \rho^2\phi, \rho^3\phi.$$
For instance, the element $\rho\phi$ meant "first flip the square about the diagonal, then rotate it", reading right-to-left.

   (a) Show by drawing pictures that $\rho\phi = \phi\rho^3$.

   (b) Compute the multiplication table for D_4, using the ordering on the elements given above. (See our homepage for a pdf to fill in, if you'd like). The first part of this problem should make your calculations easier.

   (c) Draw the Cayley graph for D_4 using the generators ρ and ϕ. Recall that the Cayley graph will have vertices labeled by the group elements and an arrow from any vertex $a \in D_4$ to the vertices $\rho * a$ and $\phi * a$.

3. Let G be a group. Elements $a, b \in G$ are *conjugate* if there exists $c \in G$ such that $c^{-1}ac = b$. It turns out that conjugacy is an equivalence relation (reflexive, symmetric, and transitive—again, Wikipedia is a good reference here). The Sage function `G.conjugacy_classes_representatives()` gives a representative from each (conjugacy) equivalence class of G. For example,

```
sage: s = SymmetricGroup(4)
sage: s.conjugacy_classes_representatives()
      [(), (1,2), (1,2)(3,4), (1,2,3), (1,2,3,4)]
```

Thus, given any element a in the symmetric group, S_4, there is some element b such that $b^{-1}ab$ is an element in the above list (and the element so determined is unique).

Replace 4, above, with other integers, and find a pattern. Report your observations. Can you describe the representatives, in general?