

1. Let $F(x, y, z) = (x + 3z, xy^2, y)$ be a vector field, and let $C(t) = (2t, t^3, t + t^2)$ be a parametrized curve with $t \in [0, 1]$. Calculate the flow of F along C in two ways: (i) integrating the flow form for F over C , and (ii) using the classical formula for $\int_C F \cdot dC$.
2. Let $F(x, y, z) = (y^2, z, 3x)$ be a vector field, and let $S(u, v) = (u, v, uv)$ be a parametrized curve with $(u, v) \in [0, 1]^2$. Calculate the flux of F through S in two ways: (i) integrating the flux form for F over S , and (ii) using the classical formula for $\int_C F \cdot \vec{n}$.
3. Thinking of each of the following 1-forms in \mathbb{R}^3 as flow forms, find the corresponding vector fields.
 - (a) $\omega = x dx + \ln(x^2 + z^2) dy + (y + xz) dz$.
 - (b) $\eta = \cos(xy) dx + \sin(yz) dz$.
4. Thinking of each of the following 2-forms in \mathbb{R}^3 as flux forms, find the corresponding vector fields.
 - (a) $\eta = -dx \wedge dy + xy dx \wedge dz$.
 - (b) $\omega = dx \wedge (y dy - (x + z^2) dz)$.
5.
 - (a) Give an concrete example of a 0-form, η , in \mathbb{R}^3 , i.e., and element $\eta \in \Omega^0 \mathbb{R}^3$.
 - (b) Interpret integration of $d\eta$ in terms of classical vector calculus. What does Stokes' theorem say in this context?
6. Let ϕ be a function on \mathbb{R}^3 , and let F be a vector field in \mathbb{R}^3 . Describe $\text{grad}(\phi)$, $\text{curl}(F)$, and $\text{div}(F)$ using flow forms, flux forms, and the exterior derivative operator, d .
7.
 - (a) State Stokes' theorem in terms of differential forms.
 - (b) Starting with $\omega \in \Omega^i \mathbb{R}^3$ for each of $i = 0, 1, 2$, give a classical/physical interpretation of Stokes' theorem.
8. What does the fact that $d^2 = 0$ say in terms of grad , curl , and div ?
9. Let ω be a k -form. When is it true that $d\omega = 0$ implies there exists a $(k - 1)$ -form, λ such that $\omega = d\lambda$? What is the implication for grad , for curl , and for div ?