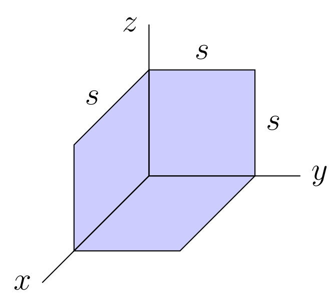
1. Under the influence of the force $F = (e^{-y} - ze^{-x})\mathbf{i} + (e^{-z} - xe^{-y})\mathbf{j} + (e^{-x} - ye^{-z})\mathbf{k}$, a partical moves along a path

$$x = \frac{1}{\ln 2}\ln(1+t), \quad y = \sin\frac{\pi t}{2}, \quad z = \frac{1-e^t}{1-e},$$

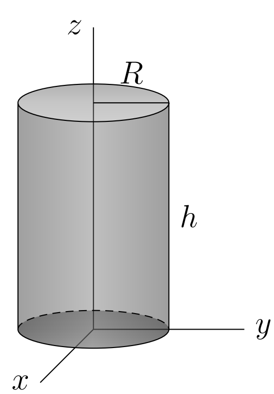
for $t \in [0,1]$. How much work does the force do on the particle?

2. For each of the following, calculate $\int\int_S F \cdot \vec{n}$.

(a) $F(x,y,z) = (x,y,z)$, and S is formed by the three squares of side length s sitting in the coordinate planes and meeting at the origin pictured below:



(b) $F(x,y,z) = \ln(x^2+y^2)\,(x\,\mathbf{i} + y\,\mathbf{j})$, and S is the cyclinder of radius R and height h pictured below:



(c) $F(x,y,z) = e^{-(x^2+y^2+z^2)}\,(x,y,z)$, and S is the sphere of radius R centered at the origin.

(d) $F(x,y,z) = f(x)\,\mathbf{i}$, and $S = \partial B_s$ where $B_s = [0,s]^3$.j

(e) $F(x,y,z) = (x, 2y, 3z)$, and S is the right cone with circular base of radius R sitting in the xy-plane, centered at the origin, and height h.

3. Consider the parametrized surface $S(u,v) = (u^2 \cos(v), u^2 \sin(v), u)$ with $(u,v) \in \mathbb{R}^2$.

   (a) Find the unit normal to S at $(u,v) = (1,0)$ using cross products.

   (b) Describe the tangent plane to S at $(u,v) = (1,0)$ by an equation of the form $a\,x + b\,y + c\,z = d$ for some $a, b, c, d \in \mathbb{R}$.

   (c) Describe the image of S by an equation for the from $f(x,y,z) = 0$. (So image(S) $= \{(x,y,z) \in \mathbb{R}^3 : f(x,y,z) = 0\}$.)

   (d) Use the gradient of the function f to verify your answer to part (a).

4. Evaluate

$$\int\int_S \frac{1}{1+4(x^2+y^2)}\, dS$$

   where S is the portion of the paraboloid $z = x^2 + y^2$ between $z = 0$ and $z = 1$.