Math 212

Quiz Let $F: \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field, let $p \in \mathbb{R}^3$, and let $v \in \mathbb{R}^3$ be a unit vector. Explain how Stokes' theorem implies that

$$\text{curl} F(p) \cdot v$$

is the circulation density (per unit area) of F about p in the plane through p perpendicular to v.

Problems

I. Think of the xy-plane as an infinite slab of charge with constant density λ (charge per unit area). Making reasonable symmetry assumptions, calculate the electric field, E, at points off of the plane.

II Consider the distribution of charge in $\mathbb{R}^3$ having density

$$\rho(x,y,z) = \begin{cases} \rho_0 & \text{if } -b \le z \le b \\ 0 & \text{if } |z| > b \end{cases}$$

for some constants ρ_0 and $b > 0$. Making reasonable symmetry assumptions, calculate the electric field, $\mathbf{E}$, at points $(x,y,z) \in \mathbb{R}^3$ with $|z| > b$.

III Consider a hollow circular cylinder of infinite length and radius R centered along the z-axis. Suppose a uniform charge on the cylinder with (constant) density λ (Coulombs/meter^2). Making reasonable symmetry assumptions, find the electric field caused by the cylinder.

IV. Consider the z-axis to be an infinitely long wire with uniform charge density λ (Coulombs/meter). Making reasonable symmetry assumptions, determine the electric field caused by the wire.

What if the wire is replaced by a cylinder of charge with density $\rho(r) = \rho_0 e^{-r/b}$ where r is the distance from the z-axis?

V Consider a spherically symmetric distribution of charge centered at the origin with density

$$\rho(r) = \begin{cases} \rho_0 & r < b \\ \rho_1 & b \leq r < 2b \\ 0 & r > 2b. \end{cases}$$

Making reasonable symmetry assumptions for the electric field, $\vec{E}$, calculate $\vec{E}$ for points further than 2b from the origin?

What is the condition on ρ_0 and ρ_1 so that $\vec{E} = 0$?
What is the total charge in that case?