Math 212

* Tomorrow's quiz

* Go over HW

$S: D \to \mathbb{R}^3$, $F: \mathbb{R}^3 \to \mathbb{R}^3$

$D \subset \mathbb{R}^2$

flow form: $\omega_F = F_1\,dx + F_2\,dy + F_3\,dz$

circulation (flow) of F along the boundary of S

$$= \oint_{\partial S} F \cdot \vec{t} = \int_{\partial S} \omega_F \overset{\text{Stokes'}}{=} \int_S d\omega_F.$$

Now, $d\omega_F = d(F_1\,dx + F_2\,dy + F_3\,dz)$

$$= (D_1F_1\,dx + D_2F_1\,dy + D_3F_1\,dz)\wedge dx$$
$$+ (D_1F_2\,dx + D_2F_2\,dy + D_3F_2\,dz)\wedge dy$$
$$+ (D_1F_3\,dx + D_2F_3\,dy + D_3F_3\,dz)\wedge dz$$

$$\Rightarrow \quad d\omega_F = (D_2F_3 - D_3F_2)\, dy\wedge dz - (D_3F_1 - D_1F_3)\, dx\wedge dz + (D_1F_2 - D_2F_1)\, dx\wedge dz$$

= flux form for the vector field

$$(D_2F_3 - D_3F_2,\ D_1F_3 - D_3F_1,\ D_1F_2 - D_2F_1)$$

Def. If $F = (F_1, F_2, F_3)$ is a vector field in $\mathbb{R}^3$, then the curl of F is the vector field

$$\operatorname{curl} F = (D_2F_3 - D_3F_2,\ D_1F_3 - D_3F_1,\ D_1F_2 - D_2F_1)$$

Another way to express the curl: Let $\nabla = (D_1, D_2, D_3)$. Then

$$\operatorname{curl} F = \nabla \times F = \det \begin{vmatrix} i & j & k \\ D_1 & D_2 & D_3 \\ F_1 & F_2 & F_3 \end{vmatrix}.$$

We've just seen that Stokes' theorem in this context says the flow of a vector field F along the boundary, ∂S, equals the flux of the curl of F through S:

$$\int_{\partial S} F \cdot \vec{t} = \int_S \operatorname{curl} F \cdot \vec{n}$$

or

$$\int_{\partial S} \omega_F = \int_S \omega^{\operatorname{curl} F}$$

↖ flow form for curl F