Math 212

* Return + discuss HW

Midterm on Friday

Boundaries

Recall definitions from yesterday. k-chain: $C = \sum_{s=1}^{\ell} v_s \varphi_{(s)}$ where

$\varphi_{(s)} : I^k \to \mathbb{R}^n$ (singular k-cube in $\mathbb{R}^n$)

number

If $\omega \in \Omega^k \mathbb{R}^n$, then $\int_C \omega = \int_{\Sigma v \varphi_{(s)}} \omega := \sum_s v_s \int_{\varphi_{(s)}} \omega$.

standard k-cube: $\Delta^k : I^k \to \mathbb{R}^k$
$\qquad p \mapsto p$.

Def. The boundary of Δ^k is $\partial \Delta^k = \sum_{i=1}^{k} \sum_{\alpha=0}^{1} (-1)^{i+\alpha} \Delta^k_{i,\alpha}$

where $\Delta^k_{i,\alpha} : I^{k-1} \to I^k \subseteq \mathbb{R}^k$ is defined by

Set ith variable equal to α

$$\Delta^k_{i,\alpha}(x_1, \dots, x_{k-1}) = \Delta^k(x_1, \dots, x_{i-1}, \alpha, x_i, \dots, x_{k-1})$$

Further, if $\varphi : I^k \to \mathbb{R}^n$, then $\quad I^{k-1} \xrightarrow{\Delta^k_{i,\alpha}} I^k \xrightarrow{\varphi} \mathbb{R}^n$

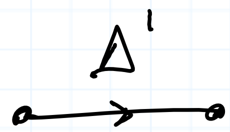
$$\partial \varphi = \varphi \circ \partial \Delta^k := \sum_{i=1}^{k} \sum_{\alpha=0}^{1} (-1)^{i+\alpha} \varphi \circ \Delta^k_{i,\alpha}$$

If $c = \sum_{v=1}^{l} v_s \varphi_{(s)}$, then

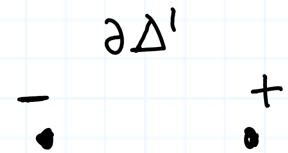
$$\partial c = \sum_{s=1}^{l} v_s \, \partial \varphi_{(s)} .$$

Examples

1. $\partial \Delta^1 = \sum_{i=1}^{1} \sum_{\alpha=0}^{1} (-1)^{i+\alpha} \Delta^1_{i,\alpha} = -\Delta^1_{i,0} + \Delta^1_{i,1}$



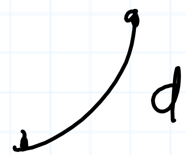
$\Delta^1_{1,0} : () \mapsto 0, \qquad \Delta^1_{1,1} : () \to 1$

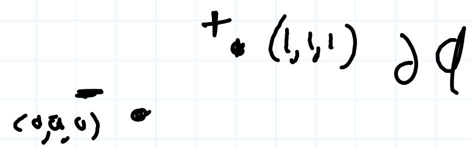


set 1st variable of Δ^1 equal to 0

2. $\varphi : I \to \mathbb{R}^3$
$\quad t \to (t, t^2, t^3)$

$$\partial \varphi = \varphi \circ \partial \Delta^1 = -\varphi \circ \Delta^1_{i,0} + \varphi \circ \Delta^1_{i,1} .$$





3. $\varphi : I^2 \to \mathbb{R}^2$

$$(r,\theta) \mapsto (r\cos(2\pi\theta), r\sin(2\pi\theta))$$

$$\partial\varphi = \sum_{i=1}^{2} \sum_{\alpha=0}^{1} (-1)^{i+\alpha} \Delta^2_{i,\alpha}$$

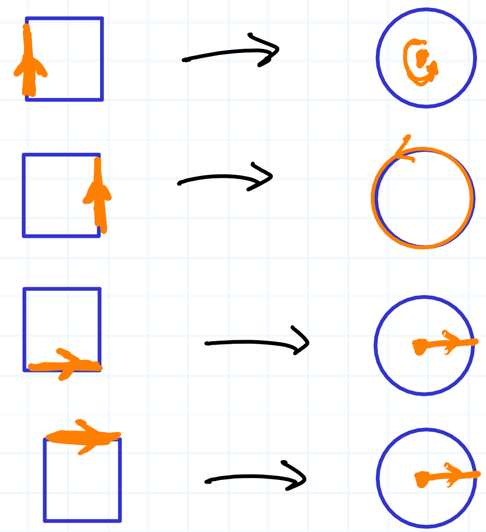
Sign mapping

$-$ $\varphi \circ \Delta^2_{1,0}(t) = \varphi(0,t) = (0,0)$

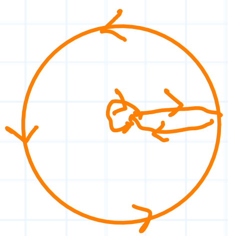
$+$ $\varphi \circ \Delta^2_{1,1}(t) = \varphi(1,t) = (\cos 2\pi t, \sin 2\pi t)$

$+$ $\varphi \circ \Delta^2_{2,0}(t) = \varphi(t,0) = (t,0)$

$-$ $\varphi \circ \Delta^2_{2,1}(t) = \varphi(t,1) = (t,0)$



[Note: $-\int_0^1 \varphi(t,1)\,dt = \int_0^1 \varphi(1-t,1)\,dt$ by change of variables $u = 1-t$, $du = -dt$]

All together:  Integration goes: