Math 212

Tomorrow's quiz: See our webpage

Review HW

Change of variables formula

$K \subseteq \mathbb{R}^n$ compact, $\mathrm{vol}(\partial K) = 0$, $\varphi : K \to \mathbb{R}^n$ cts. partials, 1-1 except on a set of volume 0, $f : \varphi(K) \to \mathbb{R}$ integrable. Then

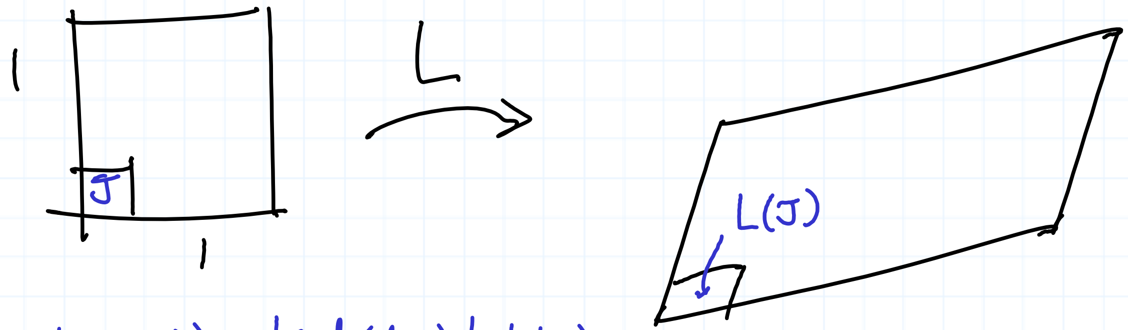
$$\int_{\varphi(K)} f = \int_K f \circ \varphi \, |\det J\varphi| .$$

$$\boxed{K} \xrightarrow{\varphi} \varphi(K) \xrightarrow{f} \mathbb{R}$$

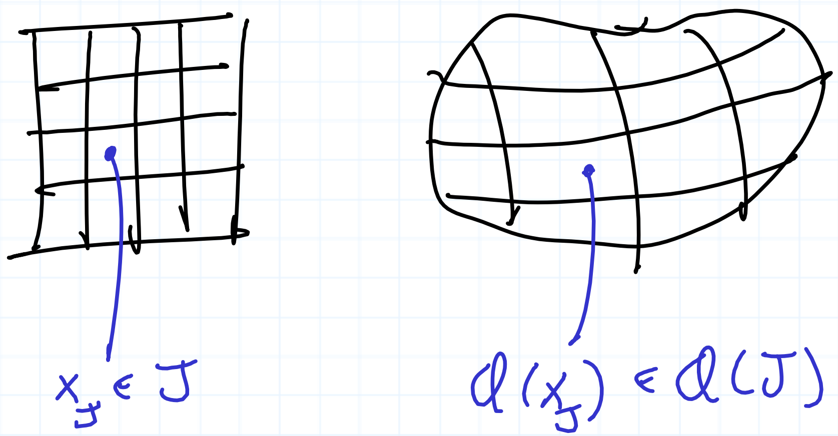
Idea of proof/ First note that if $L : \mathbb{R}^n \to \mathbb{R}^n$ is linear, then

$$\mathrm{vol}\big(L(\text{unit box})\big) = |\det A_L|$$

↖ matrix representing A.



$$\mathrm{vol}(L(J)) = |\det(A_L)|\,\mathrm{vol}(J)$$



$$\begin{aligned}
\int_{\varphi(K)} f &\approx \sum_J f[\varphi(x_J)]\,\mathrm{vol}(\varphi(J)) \\
&\approx \sum_J (f\circ\varphi)(x_J)\,|\det(J\varphi)|\,\mathrm{vol}(J) \\
&\approx \int_K f\circ\varphi\,|\det(J\varphi)|.
\end{aligned}$$

$\square$