Review for Final *Math 212*

The final exam is Tuesday, May 14, 1–4 pm., in P240A. It will be in-class and closed book/notes, etc., and will cover exactly the topics listed below.

I. Integrals. Probably the main thing to take away from this class is a variety of integrals:

- weighted volume of a bounded subset of $\mathbb{R}^n$;
- weighted k-volume (surface area) of a parametrized k-surface in $\mathbb{R}^n$;
- the integral of a k-form over a k-surface;
- the flow (work) of a vector field in $\mathbb{R}^n$ along a parametrized curve in $\mathbb{R}^n$;
- the flux of a vector field in $\mathbb{R}^n$ through a hypersurface in $\mathbb{R}^n$.

These integrals are summarized in the handout: integral summary.

Remarks.

1. You should be able to calculate each of these five types of integrals appropriately in straightforward examples.

2. The definition of weighted volume is the basis for all the other integrals. It is the one defined in terms of upper and lowers sums relative to partitions of boxes containing the set in question. You should know the precise definition.

3. In addition to integration of differential forms, you should know the basic algebra of differential forms: sums, product, exterior derivative, and pullbacks.

4. The flow of a vector field F along a curve C is defined as the integral of the flow form for F, i.e., $\int_C \omega_F$, but this integral can also be written as $\int_a^b (F \circ C) \cdot C'$. In general, the former is good for theoretical purposes and the latter is more convenient for calculations. With a little manipulation, the latter expression reveals that the formula is really calculating the length of the curve weighted by the component of F in the direction of C. You should be able to explain this (cf. Monday and Tuesday lectures, week 8).

   Similarly, for the flux of a vector field F through a surface S, we have $\int_S \omega^F = \int_D (F \circ C) \cdot (S_u \times S_v)$ (in the case $n = 3$). Manipulating the latter expression shows that the formula calculates the area of S weighted by the normal component of F. Again, you should be able to explain this (cf. Friday, week 9 and Monday, week 10).

5. Our main tools for calculating integrals are Fubini's theorem and the change of variables theorem. You should know how to use these. In particular, you should memorize the polar, cylindrical, and spherical changes of coordinates with their stretching factors.

II. Stokes' theorem, div, grad, and curl.

1. ⋆⋆ Be able to state and prove Stokes' theorem. This entails carefully reading and understanding Monday's lecture in Week 9. ⋆⋆

2. Know the definitions of div, grad, and curl and their connection to differential forms:

$$d\phi = \omega_{\nabla\phi}, \quad d\omega_F = \omega^{\nabla\times F}, \quad d\omega^F = (\nabla\cdot F)\,dx\wedge dy\wedge dz.$$

3. Know the equivalent conditions for when a vector field has a potential (i.e., when it is a gradient vector field) or for when it has a vector potential (i.e., when it is the curl of a vector field). See the notes: Monday, Week 11.

4. Know how to state Stokes' theorem in terms of div, grad, and curl. (See the integral handout referenced above.)

5. Be able to use Stokes theorem to give the intuitive geometric meaning of div, grad, and curl. See the intuition handout.

III. Maxwell's equations.

1. Memorize Maxwell's equations.

2. Know how to use Maxwell's equations to calculate the electric field about some symmetric distribution of charge.

3. Explain how Maxwell's equations suggest the existence of electromagnetic waves traveling through empty space (Tuesday, Week 12, lecture 45 and HW10). This will require memorizing one arcane formula involving the curl of the curl of a vector field.

Practice problems. In addition to the definitions and explanations your requested above, you should look over the practice problems we did during the last couple weeks of class.