HW 5, due Friday, March 8 *Math 212*

1. Compute the following and put into standard (alphabetical) form, e.g., write $-2yz\,dx\wedge dy\wedge dz$ instead of $2yz\,dz\wedge dy\wedge dx$.

   (a) $3\omega-2\lambda$ where $\omega=x^2\,dx\wedge dy+(3xy-z)\,dz\wedge dy$ and $\lambda=(x-y)\,dx\wedge dz-z\,dy\wedge dz$.

   (b) $(x\,dx+y\,dy+z\,dz)\wedge(z\,dx-2\,dz)$.

   (c) $(z\,dx-2\,dz)\wedge(x\,dx+y\,dy+z\,dz)$.

   (d) $(xy^2\,dx\wedge dz)\wedge(dx+z\,dy)$.

   (e) $(dx+z\,dy)\wedge(xy^2\,dx\wedge dz)$.

   (f) $d((xy^2-yz)\,dx\wedge dy+y^2\,dx\wedge dz)$.

   (g) $d(y\,dz)$ and $d(x\,dx)$.

   (h) $d(\cos(x))$ and $d(\cos(xy))$.

   (i) $d^2(\cos(xy)):=d(d(\cos(xy)))$.

   (j) $d(x)$ and $d^2(x)$.

   (k) $d(x^2y+3yz)$ and $d^2(x^2y+3yz)$.

   (l) Let $\omega=x^2y\,dz$ and $\lambda=(xz+yt)\,dx\wedge dt$. Compute $d(\omega\wedge\lambda)$.

   (m) With ω and λ as in the previous problem, compute $d\omega\wedge\lambda$ and $\omega\wedge d\lambda$. Compare with the previous problem. What is the general theorem (no proof required)?

2. For each of the following functions Φ with domain D and differential form ω,

   (i) compute $\Phi^*\omega$ directly from the definition of the pullback;

   (ii) compute $\Phi^*\omega$ using determinants of submatrices of $J\Phi$;

   (iii) compute the integral $\int_\Phi \omega$

   (a) $\Phi(u,v)=(u-v,u+v,u^2-v^2)$; $D=[0,1]^2$; $\omega=x^2\,dx\wedge dy$.

   (b) $\Phi(u,v)=(v,u,u^2+v^2,u^2v^3)$; $D=[0,1]^2$; $\omega=y\,dx\wedge dy+2z\,dy\wedge dz$ (with coordinates x,y,z,t on $\mathbb{R}^4$).

   (c) $\Phi(t)=(t,t^2)$, $D=[0,1]$; $\omega=y\,dx+x\,dy$.

3. Consider the 0-form $\omega=f(x,y,z)=x^2+y^2+z^2$ and the 0-surface $\Phi(())=(1,2,3)$. Compute $\int_\Phi \omega$.