Typesetting row reductions and solution spaces

Matrices. The following LateX code

```
\[
\left(
\begin{array}{rrr|r}
1 & 1 & -8 & 6 \\
2 & 3 & -21 & 14 \\
-1 & 0 & 4 & -5
\end{array}
\right)
\]
```

produces the augmented matrix

$$\left(\begin{array}{rrr|r}1 & 1 & -8 & 6 \\ 2 & 3 & -21 & 14 \\ -1 & 0 & 4 & -5\end{array}\right)$$

The part of the code that reads `{rrr|r}` says that the four entries in each row should be right-justified and that there should be a line between the third and fourth columns. To center or left-justify any column, use `c` or `l`, respectively, in place of any `r`.

Here are a couple more examples of matrices. This code

```
\[
\left(\begin{array}{rr}
1 & 3 \\
0 & 5 \\
2 & 4
\end{array}\right)
\qquad
\left(\begin{array}{ccc}
x & x^2 + x + 2 & 2 \, x^3 + 5 \, x^2 \\
3+x^{12} & 7 \, x^4 & 4
\end{array}\right)
\]
\end{array}\right)
```

produces the following:

$$\left(\begin{array}{rr}1 & 3 \\ 0 & 5 \\ 2 & 4\end{array}\right) \qquad \left(\begin{array}{ccc}x & x^2 + x + 2 & 2 \, x^3 + 5 \, x^2 \\ 3+x^{12} & 7 \, x^4 & 4\end{array}\right)$$

The `\qquad` adds horizontal space (other choices are `\quad` for half the extra space of `\qquad` or `\vspace{2cm}` for 2 centimeters of space, etc.).

The align environment. The following code

```
\begin{align*}
2x - 5y &=  8 \\
3x + 9y &=  -12\\
4x + 12y&= 7
\end{align*}
```

results in

$$\begin{aligned} 2x - 5y &= 8 \\ 3x + 9y &= -12 \\ 4x + 12y &= 7 \end{aligned}$$

The ampersand gives the point of alignment, and the `\\` gives a new line. Note that the last equation in the code does not contain `\\`.

Here is another example of the use of the align environment:

```
\begin{align*}
  x +\frac{1}{x} + \frac{x}{1+x} &= \frac{x^2+1}{x} + \frac{x}{1+x}\\[8pt]
  &= \frac{(x^2+1)(1+x)+x^2}{x(x+1)}\\[8pt]
  &= \frac{1+x+2x^2+x^3}{x(x+1)}.
\end{align*}
```

$$\begin{aligned} x +\frac{1}{x} + \frac{x}{1+x} &= \frac{x^2+1}{x} + \frac{x}{1+x}\\[8pt] &= \frac{(x^2+1)(1+x)+x^2}{x(x+1)}\\[8pt] &= \frac{1+x+2x^2+x^3}{x(x+1)}. \end{aligned}$$

Here, I have used `\\[8pt]` instead of `\\` in order to add 8 points of space between the lines. (Instead of `pt`, one could use `cm` or `mm` or `in`, etc.)

Typesetting row reduction. For typesetting row reduction, I use the align environment along with the `\xrightarrow` command, as can be seen below:

```latex
\begin{align*}
\left(
\begin{array}{rrr|r}
1 & 1 & -8 & 6 \\
2 & 3 & -21 & 14 \\
-1 & 0 & 4 & -5
\end{array}
\right)
% the alignment point is set here
&\xrightarrow[r_3\to r_3+r_1]{r_2\to r_2-2r_1}
\left(
\begin{array}{rrr|r}
1 & 1 & -8 & 6 \\
0 & 1 & -5 & 2 \\
0 & 1 & -4 & 1
\end{array}
\right)
\xrightarrow{r_3\to r_3-r_2}
\left(
\begin{array}{rrr|r}
1 & 1 & -8 & 6 \\
0 & 1 & -5 & 2 \\
0 & 0 & 1 & -1
\end{array}
\right)
\\[10pt]
&\xrightarrow{r_1\to r_1-r_2}
\left(
\begin{array}{rrr|r}
1 & 0 & -3 & 4 \\
0 & 1 & -5 & 2 \\
0 & 0 & 1 & -1
\end{array}
\right)
\xrightarrow[r_1\to r_1+3r_3]{r_2\to r_2+5r_3}
\left(
\begin{array}{rrr|r}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & -3 \\
0 & 0 & 1 & -1
\end{array}
\right).
\end{align*}
```

$$
\begin{aligned}
\left(\begin{array}{rrr|r} 1 & 1 & -8 & 6 \\ 2 & 3 & -21 & 14 \\ -1 & 0 & 4 & -5 \end{array}\right)
&\xrightarrow[r_3\to r_3+r_1]{r_2\to r_2-2r_1}
\left(\begin{array}{rrr|r} 1 & 1 & -8 & 6 \\ 0 & 1 & -5 & 2 \\ 0 & 1 & -4 & 1 \end{array}\right)
\xrightarrow{r_3\to r_3-r_2}
\left(\begin{array}{rrr|r} 1 & 1 & -8 & 6 \\ 0 & 1 & -5 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right)
\\[10pt]
&\xrightarrow{r_1\to r_1-r_2}
\left(\begin{array}{rrr|r} 1 & 0 & -3 & 4 \\ 0 & 1 & -5 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right)
\xrightarrow[r_1\to r_1+3r_3]{r_2\to r_2+5r_3}
\left(\begin{array}{rrr|r} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -1 \end{array}\right).
\end{aligned}
$$

Typesetting solution sets. Consider the linear system of equations

$$\begin{aligned} x_1 + 2x_2 \qquad\qquad &= 1 \\ x_3 + 3x_4 \quad &= 0 \\ x_5 &= 3. \end{aligned}$$

Typesetting for the above is

```
\[
  \sysdelim..
  \systeme{x_1+2x_2=1,x_3+3x_4=0,x_5=3.}
\]
```

which requires

```
\usepackage{systeme}
```

in the preamble to the LaTeX file.

The matrix corresponding to this system of equations is already in reduced row echelon form. So we can just read off the solutions. First, we solve for the pivot variables:

$$\begin{aligned} x_1 &= 1 - 2x_2 \\ x_3 &= -3x_4 \\ x_5 &= 3. \end{aligned}$$

Code:

```
\begin{align*}
x_1&= 1-2x_2\\
x_3&= -3x_4\\
x_5&= 3.
\end{align*}
```

The *parametric form* for the solution space is

$$\left\{ (1-2x_2, x_2, -3x_4, x_4, 3) : x_2, x_4 \in \mathbb{R} \right\}.$$

Code:

```
\[
  \left\{ (1-2x_2,x_2,-3x_4,x_4,3): x_2,x_4\in\R \right\}.
\]
```

The *vector form* for the solution space is

$$
\left\{
\left(\begin{array}{c}
1\\0\\0\\0\\3
\end{array} \right)
+ x_2
\left(\begin{array}{r}
-2\\1\\0\\0\\0
\end{array} \right)
+ x_4
\left(\begin{array}{r}
0\\0\\-3\\1\\0
\end{array} \right)
: x_2,x_4\in\mathbb{R}
\right\}.
$$

Code:

```
\[
\left\{
  \left(\begin{array}{c}
      1\\0\\0\\0\\3
  \end{array} \right)
  +
  x_2
  \left(\begin{array}{r}
    -2\\1\\0\\0\\0
  \end{array} \right)
  +
  x_4
  \left(\begin{array}{r}
    0\\0\\-3\\1\\0
  \end{array} \right)
  :
  x_2,x_4\in\R
\right\}.
\]
```