Math 201 Homework for Friday, Week 10

Due: Friday, November 12.

Problem 1. For each of the following matrices $A \in M_{n\times n}(F)$

(i) Determine all eigenvalues of $A$.

(ii) For each eigenvalue $\lambda$ of $A$, find the set of eigenvectors corresponding to $\lambda$.

(iii) If possible, find a basis for $F^n$ consisting of eigenvectors of $A$.

(iv) If successful in finding such a basis, determine an invertible matrix $P$ and a diagonal matrix $D$ such that $A = PDP^{-1}$.

(a) $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$ for $F = \mathbb{R}$.

(b) $A = \begin{pmatrix} 0 & -2 & -3 \\ -1 & 1 & -1 \\ 2 & 2 & 5 \end{pmatrix}$ for $F = \mathbb{R}$.

(c) $A = \begin{pmatrix} 7 & -5 \\ 10 & -7 \end{pmatrix}$ for $F = \mathbb{R}$.

(d) $A = \begin{pmatrix} 7 & -5 \\ 10 & -7 \end{pmatrix}$ for $F = \mathbb{C}$.

(e) $A = \begin{pmatrix} 2 & 0 & -1 \\ 4 & 1 & -4 \\ 2 & 0 & -1 \end{pmatrix}$ for $F = \mathbb{R}$.

Problem 2. Let $f\colon V \to V$ be a linear transformation. For a positive integer $m$, we define $f^m$ inductively as $f \circ f^{m-1}$. Prove that if $\lambda$ is an eigenvalue for $f$, then $\lambda^m$ is an eigenvalue for $f^m$.

Problem 3. Let $T\colon M_{n\times n}(\mathbb{R}) \to M_{n\times n}(\mathbb{R})$ defined as $T(A) = A^t$ (taking the transpose). One can prove that $T$ is a linear transformation.

(a) Show that the only eigenvalues of $T$ are 1 and -1. (*Hint:* Problem 2 might help.)

(b) For $n = 2$, describe the eigenvectors corresponding to each eigenvalue.

(c) Find an ordered basis $\mathcal{B}$ for $M_{2\times 2}(\mathbb{R})$ such that the matrix that represents $T$ with respect to $\mathcal{B}$ is diagonal.