Math 201 Homework for Tuesday, Week 8

Due: Tuesday, October 26.

Problem 1. For each of the following matrices, use the algorithm from class to determine whether they have inverses, and if so, find the inverse. Show your work (i.e., the row reduction). (*Pointer:* as with many linear algebra problems, it's easy to make arithmetic mistakes, but it's also easy to check your answer!)

$$\text{(a)} \quad \begin{pmatrix} 0 & 2 & 4 \\ 2 & 4 & 2 \\ 3 & 3 & 1 \end{pmatrix} \qquad\qquad \text{(b)} \quad \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & 5 & 4 \end{pmatrix}.$$

Problem 2. Let A and B be $n \times n$ matrices such that AB is invertible.

(a) Prove that A and B are invertible. *Hint:* Use rank.

(b) Give an example to show that A and B of arbitrary dimensions need not be invertible if AB is invertible.

Problem 3. Given m in $\mathbb{R}$, consider the line L in $\mathbb{R}^2$ given by those points (x, y) that satisfy $y = mx$. (If you recall from high school, this is the line through the origin of slope m.) Let $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection of $\mathbb{R}^2$ about L. Geometrically, $f(x, y)$ is the point obtained by taking the mirror image of (x, y) across L, so that the line segment connecting (x, y) and $f(x, y)$ is bisected perpendicularly by L.

One can prove geometrically (but you don't have to do that here), that f is a linear transformation. The goal of this problem is to find a closed formula for f (without having to use any crazy trigonometry).

(a) What are $f(1, m)$ and $f(m, -1)$?

(Hint: note that $(m, -1)$ is perpendicular to L, you don't have to prove this for the homework, although you might want to figure out why.)

(b) Prove that $\{(1, m), (m, -1)\}$ is a basis for $\mathbb{R}^2$.

(c) Compute the matrix for f with respect to the ordered basis $\langle (1, m), (m, -1) \rangle$ for the domain and the codomain. Then use the change of basis result to compute the matrix for f with respect to the standard basis for the domain and the codomain. Conclude by giving a closed formula for f.

(d) Explain why your result makes sense in the cases $m = 0$ and $m = 1$.