Math 201 Homework for Tuesday, Week 5

Due: Tuesday, September 28.

Problem 1. Let A be an $m \times n$ matrix with i, j-th entry A_{ij}. The *tranpose* of A, denoted A^T, is the $n \times m$ matrix with i, j-th entry A_{ji}: the i-th row of A^T is the i-th column of A. Thus, for example,

$$\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}^T = \begin{pmatrix} a & c & e \\ b & d & f \end{pmatrix}.$$

A matrix A is *skew symmetric* if $A^T = -A$ (notice the minus sign!).

Let W be the set of 3×3 skew symmetric matrices over $\mathbb{R}$.

(a) Prove that W is a subspace of the vector space of all 3×3 matrices over $\mathbb{R}$.

(b) Give a basis for W.

(c) What is $\dim(W)$?

Problem 2. Define the following matrix over the real numbers:

$$M = \begin{pmatrix} -14 & 56 & 40 & 92 \\ 6 & -24 & -17 & -39 \\ 8 & -32 & -23 & -53 \\ -1 & 4 & 3 & 7 \end{pmatrix}.$$

(a) What is the reduced echelon form for M? (You do not need to show your work for this.)

(b) Compute (i) a basis for the row space of M and (ii) a basis for the column space of M *using the algorithm presented in class* on Friday of Week 4. (There is a unique solution if you use the algorithm.)

Problem 3.

(a) Prove that there exists a linear transformation $f\colon \mathbb{R}^2 \to \mathbb{R}^3$ such that $f(2,1) = (0,-1,3)$ and $f(-1,2) = (1,0,-4)$. What is $f(5,10)$?

(b) Is there a linear transformation $f\colon \mathbb{R}^3 \to \mathbb{R}^2$ such that $f(1,2,1) = (2,3)$, $f(3,1,4) = (6,2)$ and $f(7,-1,10) = (10,1)$? Explain your reasoning.