Math 201 Homework for Week 3, Tuesday

Due: Tuesday, September 14.

Problem 1. In each of the following:

- Determine whether the given vector v is in the span of the set S.
- If v is in the span of S, then explicitly write v as a linear combination of the vectors in S.

(a) $V = \mathcal{P}_3(\mathbb{Q})$, $v = x^3 + 8x^2 + 7x - 18$,
$S = \{x^3 + 3x - 2, x^3 + 4x^2 - x + 2, x^2 - 2x + 3\}$.

(b) $V = M_{2\times 2}(\mathbb{R})$, $v = \begin{pmatrix} -1 & 3 \\ 2 & 4 \end{pmatrix}$,

$$S = \left\{ \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}, \begin{pmatrix} -1 & 2 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix} \right\}.$$

Definition. Let S be a subset of a vector space V. We say S *generates* V if $\mathrm{Span}(S) = V$.

Problem 2. Let F be a field and consider $S = \{(1,1,0), (1,0,1), (0,1,1)\} \subseteq F^3$.

(a) Prove that if $F = \mathbb{Q}$, then S generates $\mathbb{Q}^3$.

(b) Prove that if $F = \mathbb{F}_2$, then S does not generate $\mathbb{F}_2^3$.

Problem 3. Let V be a vector space over a field F, and let S_1 and S_2 be two subsets of V.

(a) Prove that $\mathrm{Span}(S_1 \cap S_2) \subseteq \mathrm{Span}(S_1) \cap \mathrm{Span}(S_2)$.

(b) Give an example in which $\mathrm{Span}(S_1 \cap S_2)$ and $\mathrm{Span}(S_1) \cap \mathrm{Span}(S_2)$ are equal, and one in which they are not equal.