Math 201 Homework for Friday, Week 3

Due: Friday, September 17.

PROBLEM 1. Determine whether the following sets are linearly dependent or linearly independent. If they are linearly dependent, find a subset that is linearly independent and has the same span.

(a) $\left\{ \begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \right\}$ in $M_{3\times 2}(\mathbb{Q})$.

(b) $\{(1,3,2),(2,-1,2),(1,2,4)\}$ in $\mathbb{R}^3$.

(c) $\{(1,1,0),(1,0,1),(0,1,1)\}$ in $(\mathbb{F}_2)^3$ (recall that $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z}$, the field with two elements).

PROBLEM 2. Let V be a vector space over $\mathbb{R}$. Let u and v be distinct vectors in V. Prove that $\{u,v\}$ is linearly independent if and only if $\{u+v, u-v\}$ is linearly independent.

BONUS: Would the same be true over $\mathbb{F}_2$?

PROBLEM 3.

(a) For any field F, we have defined the vector space F^n of n-tuples with components in F. List all elements of (i) F^2 and (ii) F^3 in the case that $F = \mathbb{F}_2$.

(b) Let $S = \{u_1, ..., u_n\}$ be a set of linearly independent vectors in a vector space over $\mathbb{F}_2$. How many elements are in $\text{Span}(S)$? Justify your solution.