Math 201 Homework for Tuesday, Week 2

Due: upload your work to Gradescope by 3 p.m. on Tuesday, September 7.

PROBLEM 1. Let L be the line in $\mathbb{R}^3$ passing through the points $(1, 1, 1)$ and $(2, 7, 4)$.

(a) Find a system of two linear equations whose solution set is L. Show your work. (*Hint*: This will likely involve solving a system of linear equations.)

(b) Give a parametrization of L (you should only need one parameter).

PROBLEM 2. Let H be the plane in $\mathbb{R}^3$ containing the points $(1, 1, 0)$, $(1, 5, -3)$, and $(1, -2, 4)$.

(a) Find a linear equation whose solution set is H. Show your work.

(b) Give a parametrization of H.

(c) What happens if we instead consider the three points $(1, 1, 0)$, $(1, 5, -3)$ and $(1, -3, 3)$? Is there such a plane? How does the process go differently for (a) and (b)?

PROBLEM 3. Let H be the subset of vectors $(x_1, \ldots, x_n)$ in $\mathbb{R}^n$ given by the set of solutions to the equation

$$a_1x_1 + \cdots + a_nx_n = d,$$

for some constants $a_1, \ldots, a_n, d$, with at least one a_i not equal to 0. Such set is called a *hyperplane*.

(a) Prove that this set of solutions can be parametrized using $n - 1$ parameters.

(b) What do you expect the dimension of H to be? (We haven't defined dimension precisely, so use your intuition.)

(c) How many points do you expect need to be given to determine a hyperplane in $\mathbb{R}^n$?