Math 201 Homework for Week 2, Friday

PROBLEM 1. Let $V = \mathbb{R}^2$. For the following pairs of operations, decide whether they make V into a vector space over $\mathbb{R}$. Justify your answer. In what follows, let $(x_1, x_2), (y_1, y_2) \in \mathbb{R}^2$ and $r \in \mathbb{R}$.

(a)
$$(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 y_2) \quad \text{and} \quad r \cdot (x_1, x_2) = (rx_1, x_2).$$

(b)
$$(x_1, x_2) + (y_1, y_2) = (2x_1 + 2y_1, 3x_2 + 3y_2) \quad \text{and} \quad r \cdot (x_1, x_2) = (rx_1, rx_2).$$

PROBLEM 2. Here are two templates for showing a subset W of a vector space V over a field F is a subspace:

Proof 1. First note that $\vec{0} \in W$ since __________. Hence, $W \neq \emptyset$. Next, suppose that $u, v \in W$. Then __________. Hence, $u+v \in W$. Now suppose $r \in F$ and $w \in W$. Then __________. Therefore, $r \cdot w \in W$. $\square$

Proof 2. First note that $\vec{0} \in W$ since __________. Hence, $W \neq \emptyset$. Next, suppose that $r \in F$ and $u, v \in W$. Then __________. Hence, $u + r \cdot v \in W$. $\square$

Use one of these two templates for each of the following exercises.

(a) Show that $W = \{(x, y, z) \in \mathbb{R}^3 \mid 2x - y - 3z = 0\}$ is a subspace of $\mathbb{R}^3$.

(b) Show that the set $W = \{f \colon \mathbb{R} \to \mathbb{R} : f(t) = f(-t)\}$ is a subspace of the vector space of real-valued functions of one variable. (You will need to carefully use the definitions given in Example 1.12, p. 84, of the text.)