Math 201 Homework for Friday, Week 1

Due: upload your work to Gradescope before class on Friday, September 3.

As for all Math 201 homework this semester, be sure to show your work for full credit, and please acknowledge your collaborators and tutors.

PROBLEM 1. *Calculations.* For each of the following systems of linear equations

- Find the associated augmented matrix M.
- Compute the reduced row echelon form E for M. **Show your work as in class, specifying your row operations.**
- From E determine whether there are solutions to the system. If there is a unique solution, state it. If there are infinitely many solutions, express the set of solutions in two ways: (i) parametrically, as in examples 2.4 and 2.5 in Chapter One, Section I.2, and (ii) in vector form as in Chapter One, Section I.3.

(a)

$$\begin{aligned} x - 2y + z &= 1 \\ -4x + 2y - z &= 0 \\ 3x + 3y - z &= 1. \end{aligned}$$

(b)

$$\begin{aligned} x + y + 3z &= 3 \\ -x + y + z &= -1 \\ 2x + 3y + 8z &= 4. \end{aligned}$$

(c)

$$\begin{aligned} x + y + 3z &= 3 \\ -x + y + z &= -1 \\ 2x + 3y + 8z &= 7. \end{aligned}$$

(d)

$$\begin{aligned} 2x - 2y - 3z &= -2 \\ 3x - 3y - 2z + 5w &= 7 \\ x - y - 2z - w &= -3. \end{aligned}$$

PROBLEM 2. *Some questions about conics.*

(a) Let $y = px^2 + qx + r$ be the equation of a general parabola. By solving a system of equations, find the constants p, q, and r so that the resulting parabola passes through the points $(-2, 15)$, $(1, 3)$, and $(2, 11)$.

(b) A (real) plane conic is a set of points of the form

$$C = \{(x, y) \in \mathbb{R}^2 : ax^2 + bxy + cy^2 + dx + ey + f = 0\}$$

for some constants $a, b, c, d, e, f \in \mathbb{R}$, not all zero. For example, the unit circle centered at the origin is the conic specified by taking $a = c = 1$, $b = d = e = 0$, and $f = -1$ to get the defining equation, $x^2 + y^2 - 1 = 0$. Note that defining equation of a conic is only determined up to a scalar multiple: for instance, $2x^2 + 2y^2 - 2 = 0$, the conic with $a = c = 2$, $b = d = e = 0$, and $f = -2$, also determines the unit circle centered at the origin.

How many points in the plane do you think must be given to determine a specific conic, in general? Why? (Note: You probably don't have the tools yet to rigorously answer this question. What are your thoughts?)