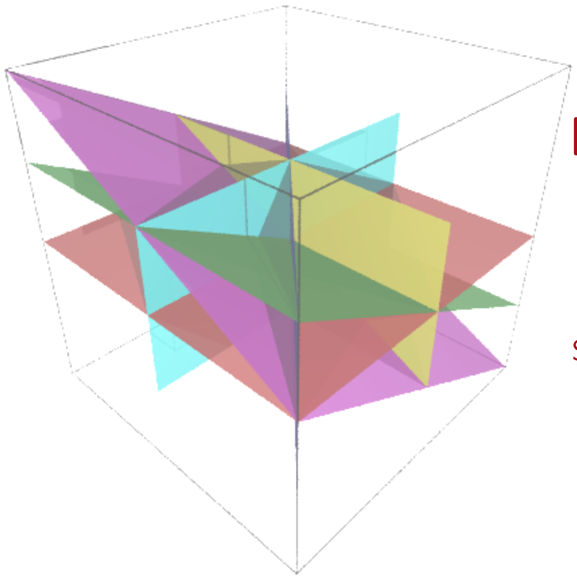

Math 201

Section F03

September 29, 2021

Colloquium

Remember to advertise the colloquium!

Range and nullspace

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Definition. The *image* or *range* of a linear function $f\colon V \to W$ is the subspace

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Remark. If $f\colon V \to W$ is a linear function, and B is a basis for V, then

$$\mathrm{im}(f) = \mathrm{Span}(f(B)).$$

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It is a subspace of V (by Proposition 2). The dimension of the kernel is called the *nullity* of f (provided it is finite-dimensional) and is denoted $\mathrm{nullity}(f)$.

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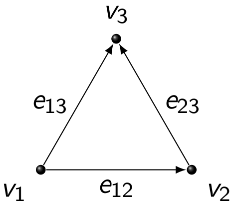
In other words,

$$\dim(\operatorname{im}(f)) + \dim(\ker(f)) = \dim V.$$

A little graph theory



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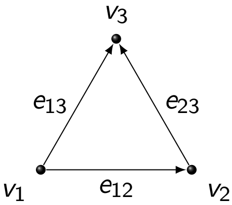

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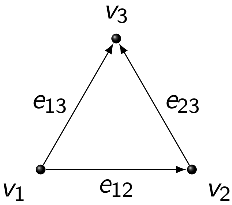

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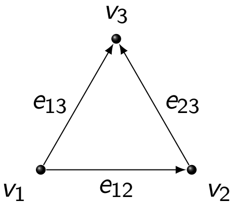

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Question: What is the kernel of ∂?