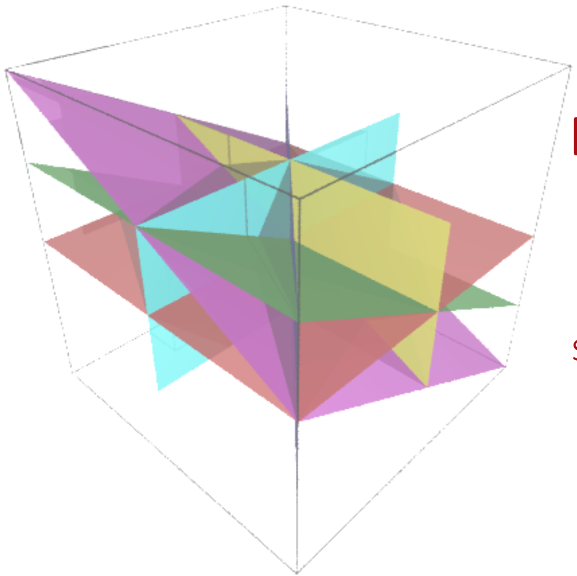

Math 201

Section F03

September 13, 2021

From last time

Definition. Let S be a nonempty subset of V. Then $v \in V$ is a *linear combination* of vectors in S if there exist $u_1, \ldots, u_n \in S$ and $a_1, \ldots, a_n \in F$ (for some n) such that

$$v = \sum_{i=1}^{n} a_i u_i = a_1 u_1 + \cdots + a_n u_n.$$

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Definition. Let S be a nonempty subset of V. The *span* of S, denoted $\mathrm{Span}(S)$, is the set of all linear combinations of elements of S. By convention $\mathrm{Span}\,\emptyset := \{0\}$, and we say that 0 is the *empty linear combination*.

Vector space of polynomials

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Is $-x^2 - 4x + 4 \in \mathrm{Span}(S)$?

Characteristic functions

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$$\chi_s\colon S \to F$$
$$t \mapsto \begin{cases} 1 & \text{if } t = s \\ 0 & \text{otherwise.} \end{cases}$$

Homogeneous linear equations

Definition. A linear equation of the form $a_1x_1 + \cdots + a_nx_n = 0$ where $a_i \in F$ is called *homogeneous*.

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Proof. Let X be the solution set to a system of homogeneous linear equations.

1. Is $0 \in X$?
2. Given $u, v \in X$ and $\lambda \in F$, is $u + \lambda v \in X$?

Generating sets of homogeneous systems

The vector form for the solution to a system of homogeneous linear equations yields a set of generators for the solution space.