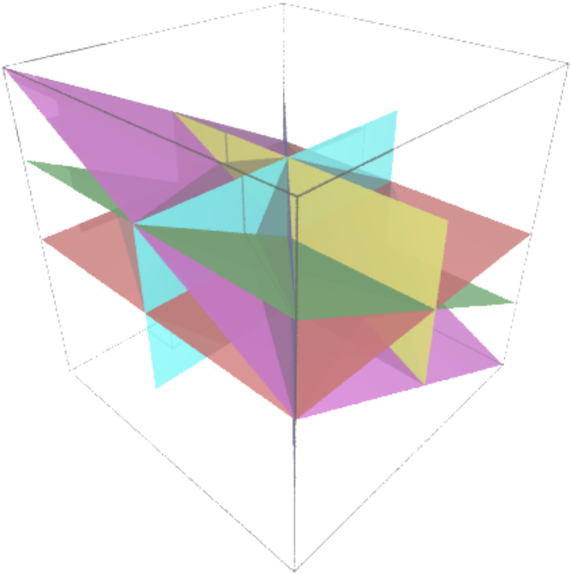

Math 201

Section F03

September 8, 2021

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Most important example of a vector space

$$F^n = \underbrace{F \times \cdots \times F}_{n \text{ times}} = \{(a_1, \ldots, a_n) : a_i \in F \text{ for } i = 1, \ldots, n\}$$

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Linear structure:

$$(a_1, \ldots, a_n) + (b_1, \ldots, b_n) := (a_1 + b_1, \ldots, a_n + b_n)$$
$$c(a_1, \ldots, a_n) := (ca_1, \ldots, ca_n)$$

for all $(a_1, \ldots, a_n), (b_1, \ldots, b_n) \in F^n$ and $c \in F$.

Matrices

The set of $m \times n$ matrices with entries in the field F:

$$M_{m \times n} := \left\{ \begin{pmatrix} a_{11} & \dots & a_{1n} \\ & \vdots & \\ a_{m1} & \dots & a_{mn} \end{pmatrix} : a_{ij} \in F \text{ for all } i, j \right\}.$$

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scalar multiplication: $(cA)_{ij} := cA_{ij}$ for all $A \in M_{m\times n}$ and $c \in F$.

Function spaces

Let S be any set, and let F be any field. Define

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$$\text{addition:} \qquad (f+g)(s) := f(s) + g(s)$$

$$\text{scalar multiplication:} \qquad (tf)(s) := t(f(s))$$

Subspaces

Definition. A subset $W \subseteq V$ of a vector space V is a *subspace* of V is it is a vector space with the operations of addition and scalar multiplication inherited from V.