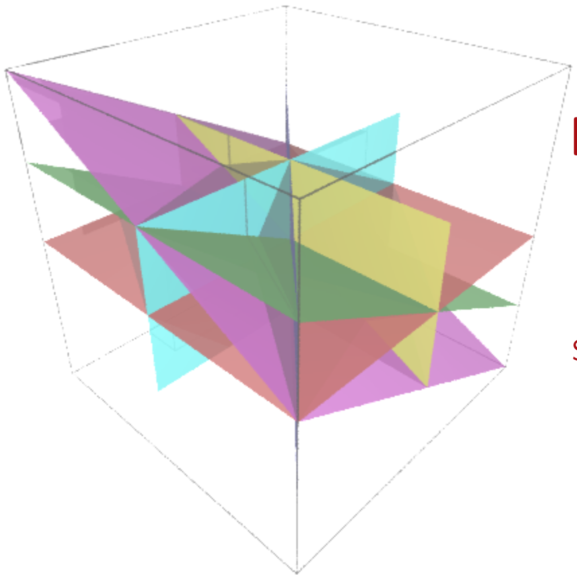

Math 201

Section F03

September 10, 2021

Linear combinations of vectors

Definition. Let S be a nonempty subset of V. Then $v \in V$ is a *linear combination* of vectors in S if there exist $u_1, \ldots, u_n \in S$ and $a_1, \ldots, a_n \in F$ (for some n) such that

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By convention $\mathrm{Span}\,\emptyset := \{0\}$, and we say that 0 is the *empty linear combination*.

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Proposition. If W_1 and W_2 are subspaces of V, so is $W_1 \cap W_2$.

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See the lecture notes for today for a proof.

Generating sets

Definition. A subset $S \subseteq V$ *generates* a subspace W if $\mathrm{Span}(S) = W$.