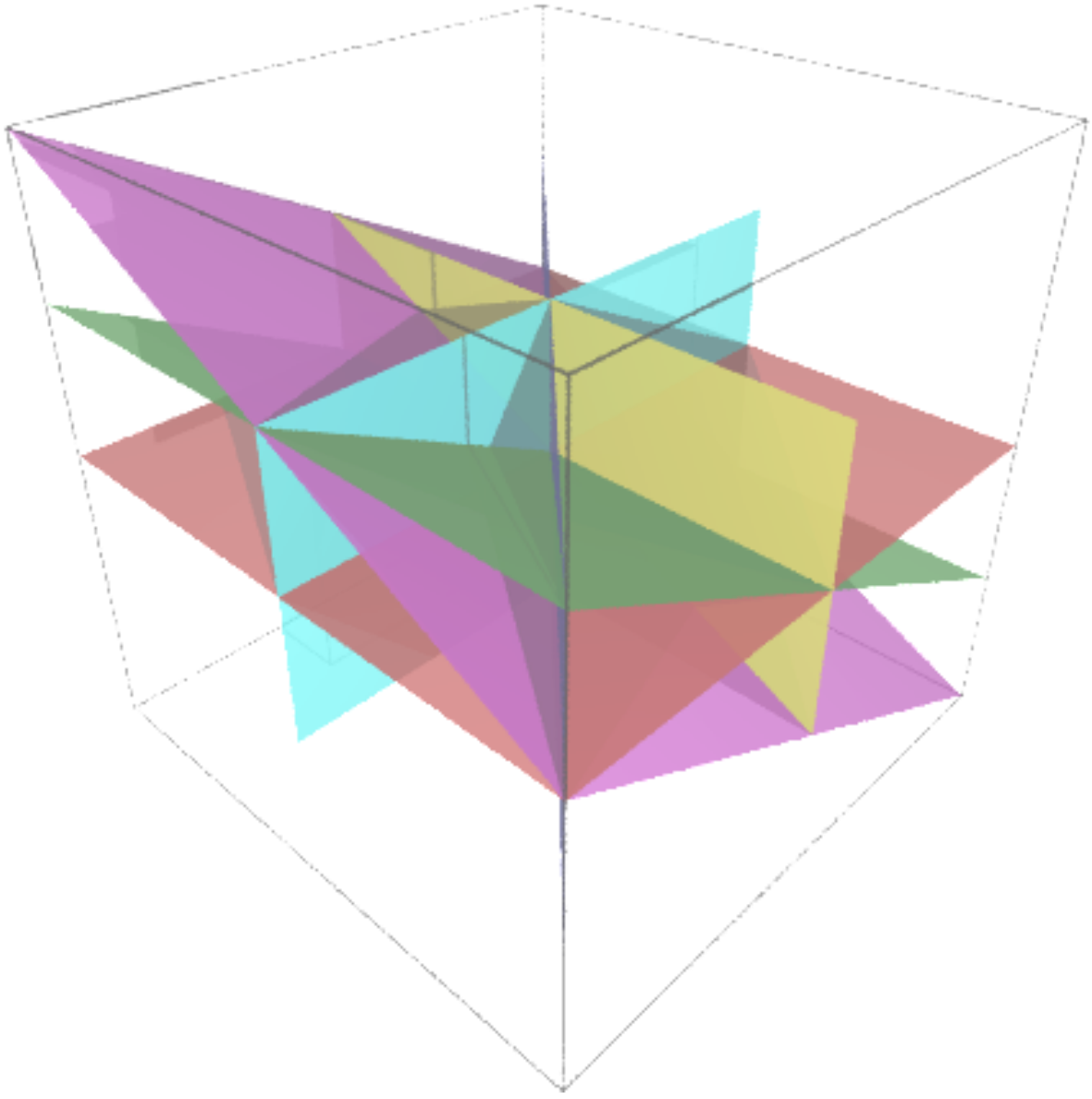


Math 201

Section F03

September 1, 2021

Goals for today

- Discuss the computation of the reduced row echelon form of a system (or matrix).
- If there is an infinite number of solutions to a system, know how to describe the solution set in two ways (which we will call the *parametric form* and *vector form*).
- Vocabulary: reduced row echelon form, *pivot* variables (or pivot columns or pivot entries), *free* variables.

Procedure for solving linear systems

1. Convert the linear system into an augmented matrix.

Procedure for solving linear systems

1. Convert the linear system into an augmented matrix.
2. Compute the reduced row echelon form of the matrix.

Procedure for solving linear systems

1. Convert the linear system into an augmented matrix.
2. Compute the reduced row echelon form of the matrix.
3. Convert the reduced matrix back into a system of equations.

Procedure for solving linear systems

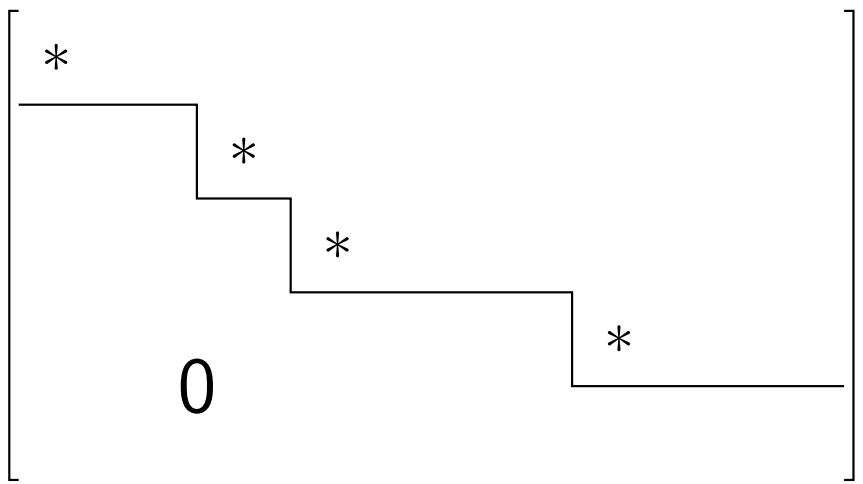
1. Convert the linear system into an augmented matrix.
2. Compute the reduced row echelon form of the matrix.
3. Convert the reduced matrix back into a system of equations.
4. Solve for the pivot variables.

Procedure for solving linear systems

1. Convert the linear system into an augmented matrix.
2. Compute the reduced row echelon form of the matrix.
3. Convert the reduced matrix back into a system of equations.
4. Solve for the pivot variables.
5. Express your solution in parametric or vector form.

Row echelon form

Definition. In each row of a system, the first variable with a nonzero coefficient is the row's leading variable. A system is in echelon form if each leading variable is to the right of the leading variable in the row above it, except for the leading variable in the first row, and any rows with all-zero coefficients are at the bottom.

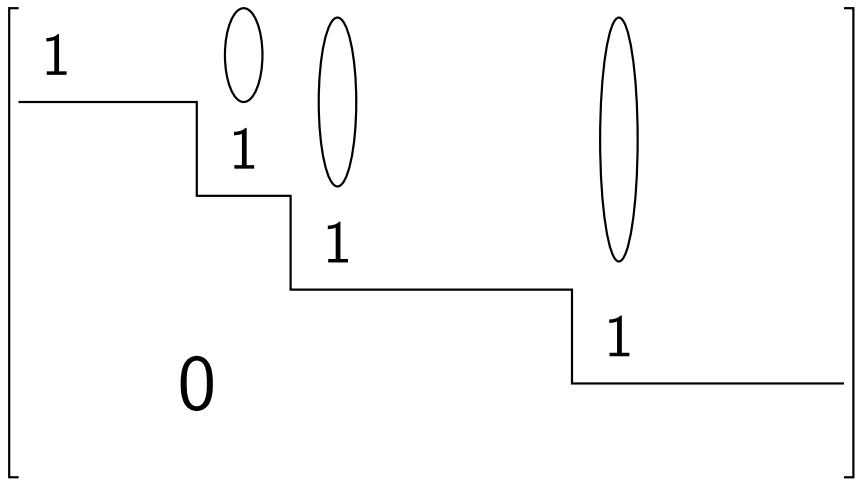


$*$ = nonzero *pivot* entry

zeros below "staircase"

reduced row echelon form

Definition. A matrix or linear system is in reduced echelon form if, in addition to being in echelon form, each leading entry is a 1 and is the only nonzero entry in its column.



pivot entries $= 1$

zeros below "staircase"

and above pivot entries

Example 1.

Find the solutions to the following linear system over the real numbers:

$$\begin{aligned} 2x_3 + 6x_4 &= 0 \\ x_1 + 2x_2 + x_3 + 3x_4 &= 1 \\ 2x_1 + 4x_2 + 3x_3 + 9x_4 + x_5 &= 5. \end{aligned}$$

Example 2

Suppose the reduced row echelon form for an augmented matrix has the form

$$\left(\begin{array}{ccccccc|c} 0 & 1 & 0 & 1 & 2 & 0 & 1 & 7 \\ 0 & 0 & 1 & 4 & -1 & 0 & 3 & -2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

Write the solution set in parametric and vector form.

Example 3

Find all parabolas $f(x) = ax^2 + bx + c$ passing through the points $(1, 4)$ and $(3, 6)$ (determine a, b, and c).

Solution

$$f(x) = \left(-1 + \tfrac{1}{3}c\right)x^2 + \left(5 - \tfrac{4}{3}c\right)x + c.$$