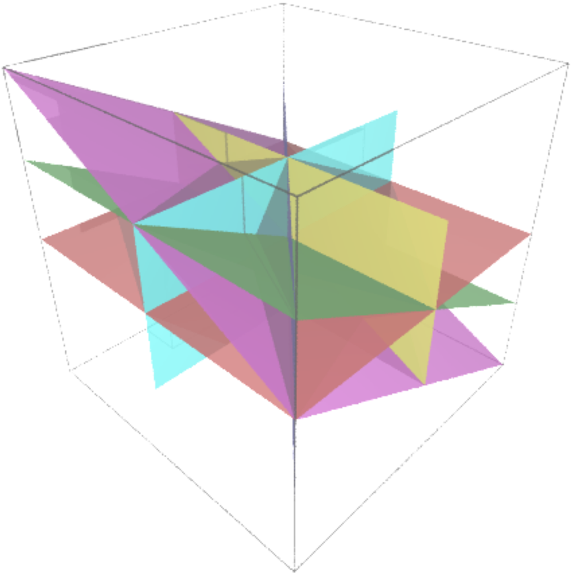

Math 201

Section F03

August 30, 2021

Administrative to-do list

- Find our course homepage:
  `https://people.reed.edu/~davidp/201`.

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- Under the Monday heading, follow the link to a to-do list for Wednesday.

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- Do the readings and practice problems for today and Wednesday.

First goal: Gaussian elimination

Our first goal is to develop an efficient procedure (algorithm) for solving systems of linear equations such as the following:

$$\begin{aligned} -w + 3x + 2y + 5z &= 5 \\ 7w + 2x - y \phantom{{}+3z} &= 1 \\ 6w + x - 3y + 3z &= 7. \end{aligned}$$

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There is a unique solution, $(1, 1)$.

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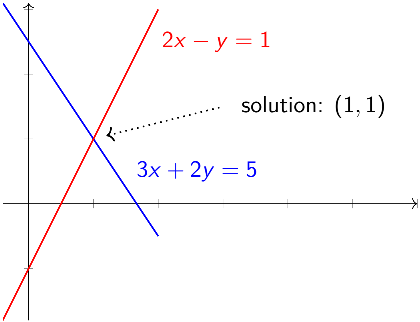

Example 2

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Solution. The second equation is a multiple of the first, so they have the same set of solutions:

$$\begin{aligned} \left\{(x, y) \in \mathbb{R}^2 : 3x + y = 2\right\} &= \left\{(x, y) \in \mathbb{R}^2 : y = -3x + 2\right\} \\ &= \{(x, -3x + 2) : x \in \mathbb{R}\} \end{aligned}$$

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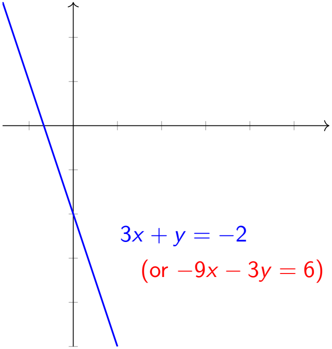

Example 3

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There are no solutions.

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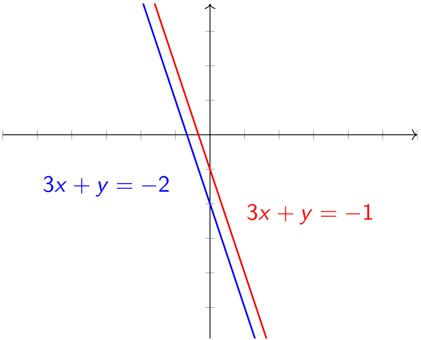
There are no solutions. The system is *inconsistent*.

Example 3

$$-9x - 3y = 6$$
$$3x + y = -1$$



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2. swap two equations
3. add a multiple of one equation to another.

Our immediate goal: develop a systematic method of using the row operations to create a new system of equations so that this new system:

- is as simple as possible, and
- has the same solution set.