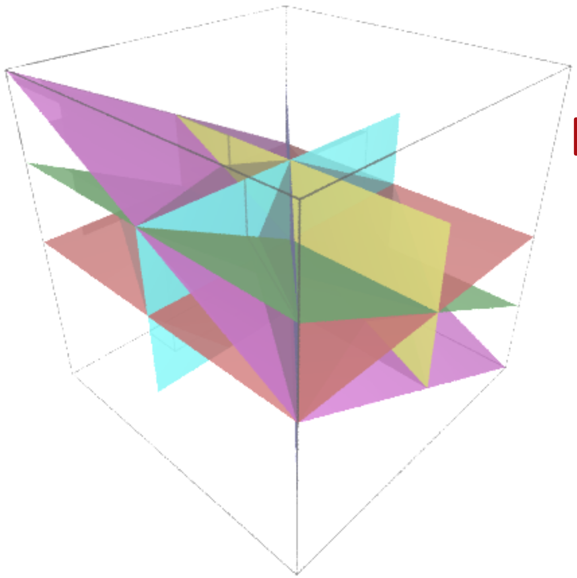

Math 201

Section F03

September 3, 2021

Real n-space

$$\mathbb{R}^n := \underbrace{\mathbb{R} \times \cdots \times \mathbb{R}}_{n\text{-factors}} := \{(x_1, \ldots, x_n) : x_i \in \mathbb{R} \text{ for } i = 1, \ldots, n\}.$$

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Addition $+ \colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$

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Scalar multiplication $\cdot \colon \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$

$$\lambda(x_1, \ldots, x_n) := (\lambda x_1, \ldots, \lambda x_n)$$

for all $(x_1, \ldots, x_n), (y_1, \ldots, y_n) \in \mathbb{R}^n$ and $\lambda \in \mathbb{R}$.

Interpretation of elements of $\mathbb{R}^n$

Elements of $\mathbb{R}^n$ can be thought of as *points* or as *arrows* (vectors).

Lines in $\mathbb{R}^n$

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$$\{p + \lambda v : \lambda \in \mathbb{R}\}$$

is the *line in* $\mathbb{R}^n$ *in the direction of* v *and passing through the point* p.

Lines in $\mathbb{R}^n$

Exercise. Show that if q is any point on the line $\{p + \lambda v : \lambda \in \mathbb{R}\}$ and w is any nonzero scalar multiple of v, then

$$\{q + \lambda w : \lambda \in \mathbb{R}\} = \{p + \lambda v : \lambda \in \mathbb{R}\}$$

Example

Give a parametrization of the line through the points $(1, 2, 0)$ and $(0, 1, 1)$.

Example

A line in $\mathbb{R}^2$ can always be expressed as the solution set to a single linear equation.

Find such an equation for the line

$$L := \{(3,2) + \lambda(1,5) : \lambda \in \mathbb{R}\}.$$

Example

A line in $\mathbb{R}^3$ *is always the solution set to a system of two linear equations.*

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A line in $\mathbb{R}^3$ is always the solution set to a system of two linear equations.

Do so for the line through the points $(1, 0, 2)$ and $(3, 1, -1)$, parametrized by

$$t \mapsto (1, 0, 2) + t\,((3, 1, -1) - (1, 0, 2)) = (1, 0, 2) + t(2, 1, -3).$$

Planes in $\mathbb{R}^n$

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Definition. Let $p, v, w \in \mathbb{R}^n$. Suppose that v and w are nonzero and that neither is a scalar multiple of the other. Then

$$\left\{ p + \lambda v + \mu w : (\lambda, \mu) \in \mathbb{R}^2 \right\}$$

is the plane in $\mathbb{R}^n$ containing p and with directions v and w.

Example

Find the plane P through the points $(0, 2, -1)$, $(4, 2, 1)$, and $(1, 0, 1)$. Describe both parametrically and as the solution set to a single linear equation.