

**MATH 113: DISCRETE STRUCTURES**  
**HOMEWORK DUE WEDNESDAY WEEK 13**

*Problem 1.* Use the Euclidean algorithm to compute the following (showing your work):

- (a)  $\gcd(20, 45)$       (b)  $\gcd(247, 299)$       (c)  $\gcd(51, 897)$ .

*Problem 2.* Use the Euclidean algorithm to compute the gcd of 198 and 168 and then use back-substitution to find integers  $m$  and  $n$  such that

$$\gcd(198, 168) = 198m + 168n.$$

Show your work. Remember to use back-substitution and not the extended Euclidean algorithm.

*Problem 3.*

- (a) Show that if  $n$  is positive integer of the form  $4k + 3$  for some integer  $k$ , then  $n$  is not a perfect square. (Hint: Suppose  $n = m^2$ . We can then write  $m = 4q + r$  for some  $r \in \{0, 1, 2, 3\}$ . Consider the remainders of the quantities  $(4q)^2$ ,  $(4q+1)^2$ ,  $(4q+2)^2$ , and  $(4q+3)^2$  upon division by 4.)
- (b) Show that no integer in the sequence

$$11, \quad 111, \quad 1111, \quad 11111, \quad \dots$$

is a perfect square. [Hint: Use the fact that  $111 \dots 1111 = 111 \dots 1108 + 3$ .]