MATH 113: DISCRETE STRUCTURES HOMEWORK DUE WEDNESDAY WEEK 10

Problem 1. You have three coins. Two of the coins are fair: when flipped they are equally likely to land heads or tails. One coin, however, is weighted somehow so that its probability of landing heads is 3/4.

- (a) Choose one of the three coins uniformly at random and flip it. What is the probability the result is heads? For your solution, number the coins 1, 2, 3 with coin 3 being the weighted one, and let A_i denote the event that coin *i* was chosen. Apply the generalized law of total probability (Theorem 177 our text).
- (b) Choose one of the three coins at random and flip it. It lands heads. What is the probability that you chose the weighted coin? (Hint: Bayes' law.)

Problem 2. Reconsider the Monty Hall problem as stated in the group problems for Friday, Week 9, but where the game show has a bias for where it places the car so that P(A) = 0.4, P(B) = 0.3, and P(C) = 0.3. (In advance of your turn on the show, suppose that you studied taped shows to determined these propensities.) As in the group problems, let M_A , M_B , and M_C denote the events that "the host opens door A", "door B", and "door C", respectively.

- (a) Show that no matter which door you pick, it makes sense to switch.
- (b) Which door should you pick?
- (c) What are your chances of eventually winning the car if you make that pick?

Note:

- » The rules for Monty are the same: if you pick the door with the car, then Monty chooses between the remaining doors each with probability 1/2, otherwise, Monty has only one choice: pick the door without the car.
- » Up to symmetry, you just need to consider three cases: (i) you pick door A and Monty picks door B, (ii) you pick door B and Monty picks door A, and (iii) you pick door B and Monty picks door C. (If the all three of P(A), P(B), and P(C) were different, there would be six cases to think about.)