MATH 113: DISCRETE STRUCTURES
HOMEWORK DUE FRIDAY WEEK 4

Problem 1. Use the binomial theorem to express 3^n as a sum of powers of two times binomial coefficients.

Problem 2. Let X be set of all subsets of size three from $\{1, \ldots, n+2\}$. For instance, if $n = 2$ we would have

$$X = \{\{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}\}.$$

In general, the number of such subsets is $|X| = \binom{n+2}{3}$. Each element of X consists of three numbers, which we list in order: $a < b < c$. For each integer b, let X_b be all subsets of $\{1, \ldots, n+2\}$ of the form $\{a, b, c\}$ for which $a < b < c$. We get a partition of X:

$$X = X_2 \amalg X_3 \amalg \cdots \amalg X_{n+1},$$

and hence

$$|X| = |X_2| + |X_3| + \cdots + |X_{n+1}|. \tag{$\star$}$$

(a) Determine (with explanation, of course) the size $|X_b|$ for $b = 2, 3, \ldots, n+1$ in terms of b and n.
(b) Equation ($\star$) becomes what identity? (In other words, replace the quantities on the left and right in Equation ($\star$) with formulas. Note: to be sure of your answer, you should check it for small n on scratch paper.)

Note. Combinatorial identities often arise from partitioning a set. On your own, you may want to consider how the Problem 1 involves a partition.