

PROBLEM 1. When is  $a \equiv b \pmod{2}$ ?  $a \equiv b \pmod{1}$ ?  $a \equiv b \pmod{0}$ ?

PROBLEM 2. What are the last two digits of  $99^{100000^{100000}+2021}$ ?

PROBLEM 3. Recall the equivalence relation from the mini-lecture: having fixed  $n \in \mathbb{Z}$ , for  $a, b \in \mathbb{Z}$ , we say  $a \sim b$  if  $a - b = kn$  for some  $k \in \mathbb{Z}$ . In other words,  $a \sim b$  if and only if  $a \equiv b \pmod{n}$ . Take  $n > 0$ , for convenience.

- (i) Show that  $\sim$  is an equivalence relation.
- (ii) State the division algorithm for integers  $a$  and  $n$ , and use it to determine the number of equivalence classes for  $\sim$ .

PROBLEM 4. Suppose  $a \equiv a' \pmod{n}$  and  $b \equiv b' \pmod{n}$ .

- (i) Prove that  $a + b \equiv a' + b' \pmod{n}$ .
- (ii) Prove that  $ab \equiv a'b' \pmod{n}$ .

PROBLEM 5. (If you have extra time.) Let  $V := \{0, 1, \dots, n-1\}$  for some positive integer  $n$ , and fix  $a \in V$ . Let  $G(a, n)$  be the directed graph with vertex set  $V$  and with an edge from  $b$  to  $c$  if  $c = b + a \pmod{n}$ . Draw this graph for various  $a$  and  $n$ , and try to deduce its general structure.