

PROBLEM 1. With your group, roll a pair of dice twelve times. Record the first roll on which you roll doubles and also the total number of doubles that you roll and report these numbers to the instructor. What is the expected number of doubles in twelve rolls? How long should it take to roll doubles? How do these numbers compare with the class's statistics?

PROBLEM 2. An airline has sold 205 tickets for a flight that can hold 200 passengers. Each ticketed person, independently, has a 5% chance of not showing up for the flight. What is the probability that more than 200 people will show up for the flight?

PROBLEM 3. If the same airline consistently oversells the flight from Problem 2 at the same rate, how many flights until we expect more ticketed passengers to show up than there are seats.

PROBLEM 4. With a binomial random variable, we run experiments independently, but there are many circumstances of interest that do not follow this pattern. One such is *sampling without replacement*: suppose we have a basket of  $N$  lottery tickets,  $K$  of which are winners. Consider a process in which you draw  $n$  of the tickets from the basket. Let  $X$  denote the number of winning tickets drawn; this is called a *hypergeometric random variable*.

Assume in this problem that  $0 \leq K, n \leq N$ .

(i) Prove that

$$P(X = k) = \frac{\binom{K}{k} \binom{N-K}{n-k}}{\binom{N}{n}}.$$

- (ii) In an election audit, a sample of machine-counted precincts are recounted by hand to check if the machine and hand audits match. Suppose there are  $N$  precincts,  $K$  of them have counting errors, we sample  $n$  precincts, and  $X$  counts the number of precincts in which errors are detected. In what sense is  $X$  a hypergeometric random variable, and what is the significance of the quantity  $P(X = 0)$ ?
- (iii) Suppose that there are machine-counting errors in 7 of 200 precincts. How many precincts must one sample in order to guarantee that there is at most a 5% chance of detecting no errors?