PROBLEM 1. Let $a, b, c \in \mathbb{Z}$ and suppose that $a|b$ and $b|c$. Prove that $a|c$. (Start by appealing to definition of divisibility to unravel the meaning of $a|b$ and $b|c$.)

PROBLEM 2. Prove that if $a|b$ and $a|c$, then $a|(mb + nc)$ for all $m, n \in \mathbb{Z}$.

PROBLEM 3. Suppose $p$ is prime and that $a$ and $k$ are positive integers. Why is it the case that if $p|a^k$, then $p^k|a^k$?

PROBLEM 4. Prove that if $p$ is a prime number, then $\sqrt{p}$ is irrational.

PROBLEM 5. Prove that a positive integer $n$ is prime if and only if it is not divisible by any prime $p$ such that $1 < p \leq \sqrt{n}$. What does this say in the case $n = 91$?

PROBLEM 6. Suppose that a positive integer $n$ has prime factorization $n = p_1^{a_1} \cdots p_k^{a_k}$ with the $p_i$ distinct primes. How many distinct positive integers are divisors of $n$?