PROBLEM 1. Find someone in your group who has taken calculus and have them assure you that

$$\frac{d}{dx}(x+1)^n = n(x+1)^{n-1}$$

and that

$$\frac{d}{dx}\sum_{k=0}^{n}\binom{n}{k}x^k = \sum_{k=1}^{n}k\binom{n}{k}x^{k-1}.$$

What identity results when you plug in $x = 1$? Can you find a proof that does not use calculus?

PROBLEM 2. Let

$$F = \{f\colon \{a,b,c,d\} \to [6] \mid f(a) \neq 1 \text{ or } f(b) \neq 2 \text{ or } f(c) \neq 3\}.$$

Determine $|F|$. (*Hint*: PIE.)

PROBLEM 3. Fix an integer $n \geq 1$ and let $V = 2^{[n]}$. Define a graph $G = (V, E)$ on the vertex set V such that for $A, B \in V$, $\{A, B\} \in E$ if and only if (1) $A \subseteq B$ or $B \subseteq A$ and (2) the larger of the two sets has exactly one more element than the smaller one.

(i) Draw G for $n = 1$, 2, and 3.
(ii) What is $|V|$?
(iii) Let $A \in V$ be a subset of $[n]$ of cardinality k. What is the degree of the vertex A? Explain.
(iv) There is a standard formula relating the degrees of the vertices to the number of edges. Use this formula to compute $|E|$.
(v) For which n does G have a closed Eulerian walk? Explain.

PROBLEM 4. Under Joyal's bijection, what is the vertebrate corresponding to the function $f\colon [8] \to [8]$ given by

n	1	2	3	4	5	6	7	8
$f(n)$	2	3	4	8	6	7	8	6

PROBLEM 5. Using the bijections given in our text, find all the Catalan structures you know corresponding to the balanced parenthesization ( ) ((( ) ( ))).