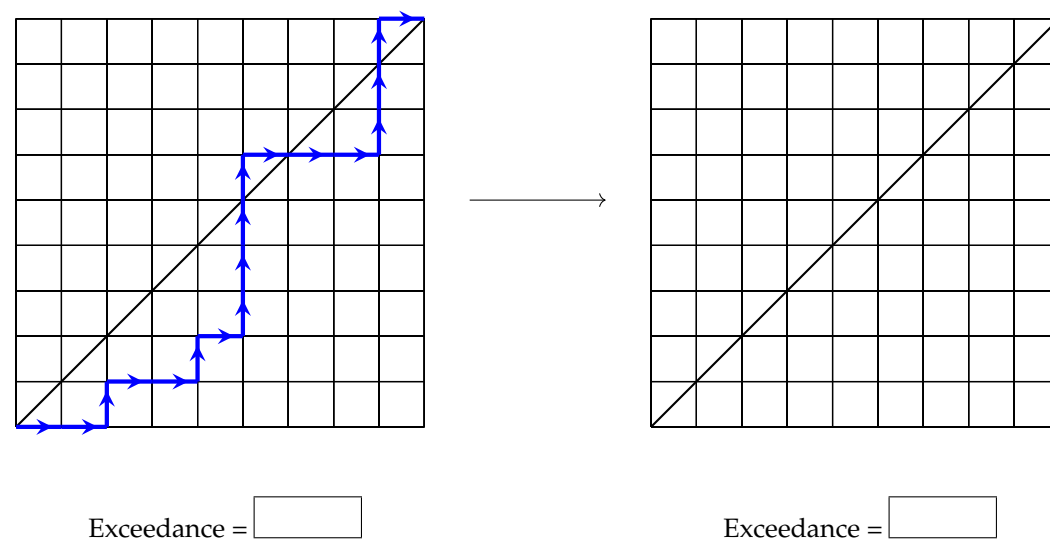
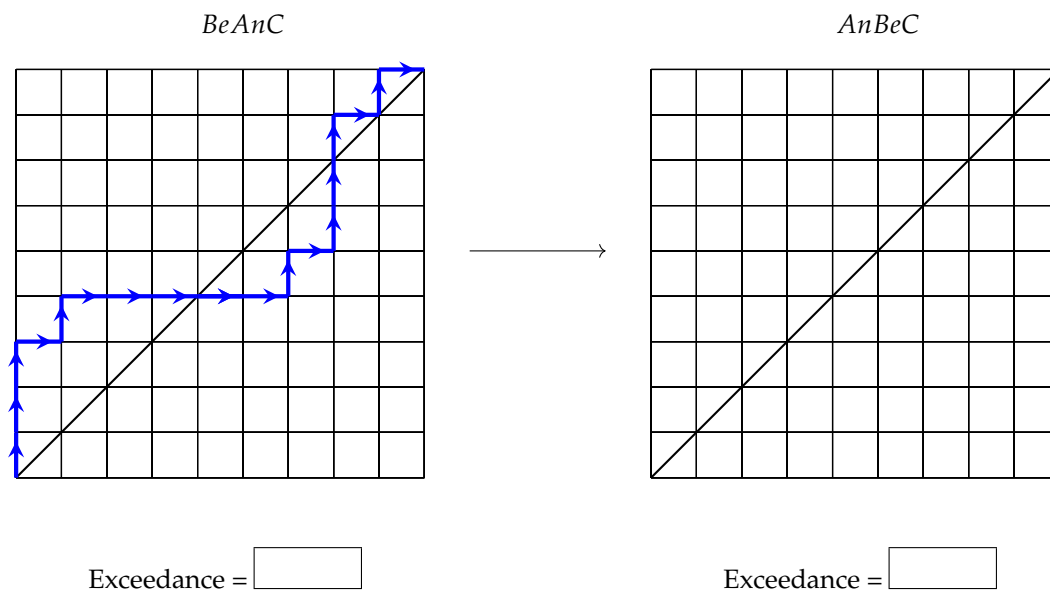
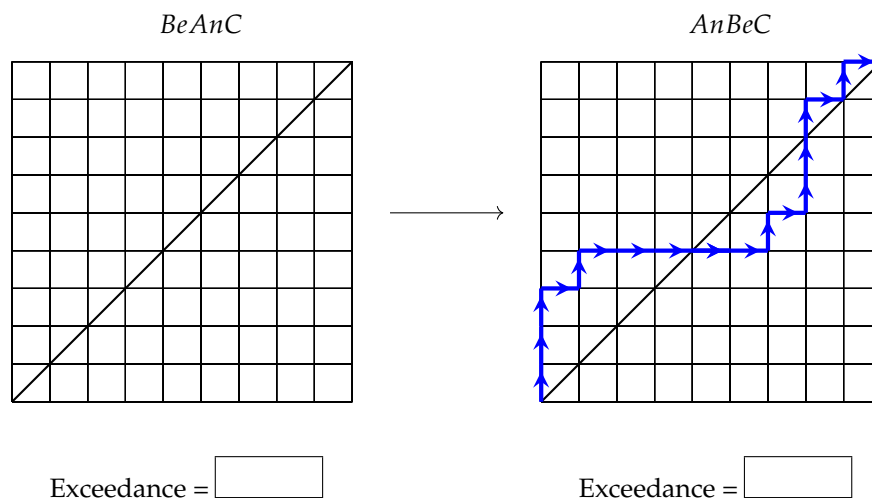


PROBLEM 1. Illustrate the bijection $E_i \rightarrow E_{i+1}$.

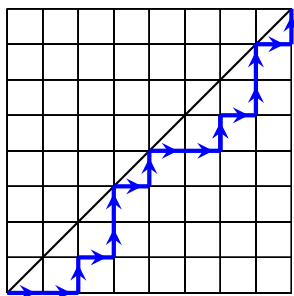


PROBLEM 2. Illustrate the inverse of the bijection $E_i \rightarrow E_{i+1}$. (Hint: something tricky occurs with A here.)



PROBLEM 3. In your reading, you saw that expressions consisting of n balanced parentheses $()$ are in bijection with Dyck paths of length $2n$, and, thus, the number of such expressions is the n -th Catalan number, C_n .

- (i) Describe the bijection between Dyck paths and balanced parentheses, and apply it to the Dyck path below.



- (ii) What is the Dyck path associated with $((()))()$?

PROBLEM 4. Use the Catalan recurrence,

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = \sum_{k=0}^n C_k C_{n-k} \quad \text{for } n \geq 0,$$

to compute the fifth Catalan number by hand.

PROBLEM 5. Explain the significance of the following sequence:

un, dos, tres, quatre, cinc, sis, set, vuit, nou, deu, ...