PROBLEM 1. A *complete graph* on n vertices, denoted K_n, has every possible edge. Draw pictures of K_3, K_4, and K_5. How many edges are there in a complete graph on n vertices? What it the maximal number of edges for a graph G with vertex set V? What is the minimal number of edges for a graph G with vertex set V?

PROBLEM 2. A graph $G = (V, E)$ is called *bipartite* if it is possible to partition V with nonempty sets as $V = A \amalg B$ such that edges only go between A and B. The *complete bipartite graph on $p + q$ vertices*, denoted $K_{p,q}$, has $|A| = p$, $|B| = q$, and all possible edges between A and B.

(i) Draw pictures of $K_{2,3}$ and $K_{3,5}$.
(ii) How many edges are in $K_{p,q}$?
(iii) If $|A| = p$ and $|B| = q$ with $A \cap B = \varnothing$, how many (not necessarily complete) bipartite graphs have vertex set $A \cup B$ with $A \amalg B$ as the specified partition?

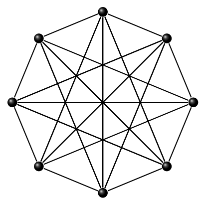
PROBLEM 3. The definition of graph isomorphism implies that isomorphic graphs have the same number of vertices and same number of edges.

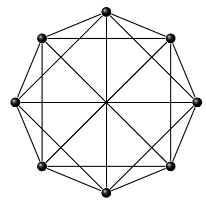
(i) Must two graphs with the same number of vertices and same number of edges be isomorphic? Prove it or find a counterexample?
(ii) The *degree sequence* of a graph is a list of its vertex degrees in non-decreasing order. Prove that graphs with the same degree sequence have the same number of edges.
(iii) Must two graphs with the same degree sequences be isomorphic? Prove it or find a counterexample.

Challenge

Challenge problems are optional and should only be attempted after completing the previous problems.

(i) Determine whether the following graphs are isomorphic.





(ii) Determine whether the graphs in any pair of the following are isomorphic.