PROBLEM 1. How many poker hands (5 cards) from a regular deck (52 cards) have at least one card from each of the four standard suits? *Hint*: Let $N_\spadesuit$ be the collection of hands containing no spades, and similarly define $N_\clubsuit$, $N_\heartsuit$, and $N_\diamondsuit$. What is the relationship between the answer to this question and $|N_\spadesuit \cup N_\clubsuit \cup N_\heartsuit \cup N_\diamondsuit|$?

PROBLEM 2. Recall that D_m denotes the number of derangements of $[m]$. How many derangements π of $[n]$ have $\pi(1) = 2$ and $\pi(2) = 1$? Fix some k such that $2 \leq k \leq n$; how many derangements π of $[n]$ have $\pi(1) = k$ and $\pi(k) = 1$?

PROBLEM 3. How many derangements π of $[n]$ have $\pi(1) = 2$ and $\pi(2) \neq 1$? Fix some k such that $2 \leq k \leq n$; how many derangements π of $[n]$ have $\pi(1) = k$ and $\pi(k) \neq 1$?

PROBLEM 4. Let D_n be the number of derangements of $[n]$. Use your answers to Problems 2 and 3 to find a formula for D_n in terms of D_{n-1} and D_{n-2}. Determine D_1 and D_2 by hand and then use your formula to determine D_n for $n = 3$, 4, 5, and 6; check that your answers match with the closed formula given in the text.