PROBLEM 1. Consider the following relations on the set \mathbb{R} of real numbers: inequality (\neq), strictly greater than (>), and less than or equal to (\leq). Determine which (if any) of the three properties of an equivalence relation these relations have:

relation	reflexivity	symmetry	transitivity	
¥				
>				•
\leq				

PROBLEM 2. Consider the relation \sim on \mathbb{R} such that $x \sim y$ if and only if x - y is an integer.

- (i) Give a formal proof (following our template) that \sim is an equivalence relation.
- (ii) Draw the real number line, choose a point, and draw that point's equivalence class. Repeat for several points.
- (iii) What does a generic element of \mathbb{R}/\sim look like? Does \mathbb{R}/\sim has a natural "shape"?

PROBLEM 3. We place two red and two black checkers on the corners of a square. Say that two configurations are equivalent if one can be rotated to the other.

- (i) Check that this is an equivalence relation.
- (ii) Draw the elements in each equivalence class.
- (iii) If ∼ is a relation on a finite set *S*, and each equivalence class has the same number *k* of elements, then the overcounting principle says the number of equivalence classes is |*S*|/*k*. Why don't these ideas apply to our problem?

PROBLEM 4. A total of *n* Americans and *n* Russians attend a meeting and sit around a round table. If Americans and Russians alternate seats, in how many ways may they be seated up to rotation? Discuss your solution in terms of an equivalence relation and equal-sized equivalence classes.

Recall that for \simeq an equivalence relation on set *X*, *X*/ \simeq is the set of equivalence classes for \simeq .

