PROBLEM 1. Use induction to show that

$$2^0 + 2^1 + 2^2 + \cdots + 2^{n-1} = 2^n - 1$$

for $n \geq 1$. Write a complete proof using the template from our text as a guide.

PROBLEM 2. Use induction to prove that the number of diagonals in a convex n-gon is $n(n-3)/2$ for $n \geq 3$.

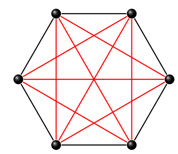

Figure 1: A hexagon has $\frac{6(6-3)}{2} = 9$ diagonals.

PROBLEM 3. There are ten pirates on a ship—conveniently named One through Ten—and they decide to use an ancient pirate method to divvy up their booty of 100 gold doubloons. Pirate One will propose a distribution and all pirates will vote. If half or more vote aye, the distribution is accepted. If not, the distribution is rejected and Pirate One is sent to Davy Jones' locker. There would then be nine pirates left, and the method continues with Pirate Two taking One's place. If Two is also forced to walk the plank, then there will be eight pirates left, and it's Pirate Three's turn, and so on. You are Pirate One. What do you propose?

Figure 2: Blackbeard the Pirate: this was published in Defoe, Daniel; Johnson, Charles (1736) "Capt. Teach alias Black-Beard" in A General History of the Lives and Adventures of the Most Famous Highwaymen, Murderers, Street-Robbers, &c. to which is added, a genuine account of the voyages and plunders of the most notorious pyrates. Interspersed with several diverting tales, and pleasant songs. And adorned with the Heads of the most remarkable Villains, curiously engraven on Copper, London: Oliver Payne, pp. plate facing p. 86 [Wikimedia Commons].

Challenge

Challenge problems are optional and should only be attempted after completing the previous problems.

Using induction, we can prove that in every gathering of Reed students, all the students have the same hair color. The formal statement is: If X is a set of n Reed students, then all the students in X have the same hair color.

We induct on the size of the set of students in the gathering. The base case of $n = 1$ is clear. So assume the result holds for some $n \geq 1$. Let X be a set of Reed students of size $n + 1$. Choose a student $A \in X$. Removing that student from X produces the set $X \setminus \{A\}$ of size n. By induction, all of these students have the same hair color H_1. Now remove a different student B from X. By induction, again, all the students in $X \setminus \{B\}$ have the same hair color H_2. Notice that $A \in X \setminus B$, and therefore has hair color H_2. Similarly, B has hair color H_1. Now for the interesting part: Let $C \in X$ be a student who has not been chosen, yet, i.e., C is neither A nor B. Since $C \in X \setminus A$, we know C's hair color is H_1. Similarly, since $C \in X \setminus B$, we know C's hair color is H_2. It follows that $H_1 = H_2$. We have accounted for every student in X and shown they have the same hair color. The result now follows by induction. What, precisely, is wrong with this argument?